MARCH 1957
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JOURNAL
OF THE
AMERICAN CONCRETE INSTITUTE

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MARCH 1957

No. 9

Vol. 28

PROCEEDINGS VOL. 53

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Technical pages of the ACI JOURNAL offer a medium for reporting what is new and interesting in concrete research, design, construction, maintenance, or manufacturing. ACI members and JOURNAL readers are urged to share their knowledge and experience by submitting papers and reports for publication in the ACI JOURNAL. Time and changing interests demand that new contributors with new ideas take their place beside the old.

●

The Technical Activities Committee selects the papers, committee reports, discussions, and other contributions for JOURNAL publication. A volunteer corps of experts in various segments of the field assists in technical appraisal of the manuscripts. Technically speaking, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

Further questions raised by TAC: will the contribution fit into a balanced publication schedule? Can it be published at reasonable expense to the Institute?

●

Contributors should furnish glossy prints or inked tracings of illustrations along with manuscripts submitted in triplicate for publication. Details of the form and physical arrangement of contributions are given in the 1955 *ACI Directory*, p. 14. Separate copies of this "publications policy" are available free on request to the Secretary.

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In ACI Journal next month

Heading the list of JOURNAL offerings in April will be the convention address of retiring Institute president FRANK KEREKES, whose retrospective glance surveys a year of outstanding growth and development for ACI.

"Proposed Revision of Specifications for Concrete Pavements and Concrete Bases (ACI 617-51)," reported by ACI COMMITTEE 617, is a companion piece to the pavement design report presented by ACI Committee 325 in the February JOURNAL. Both proposed standards have been published for purposes of discussion only. The forthcoming report deals with pavement materials, preparation of the soil foundation for pavements, form handling and removal, joint installation, and placement of reinforcement. Proportioning, materials handling, and curing are also discussed.

M. W. FERGUSON, G. L. KALOUSEK, and C. W. SMITH are authors of "Tests of a New Method for Evaluating Volume Changes of Concrete Masonry Units." Their studies suggest a more rapid procedure for measuring shrinkage of concrete block, and some work is reported on the correlation of laboratory tests with field performance. Significant behavior differences between autoclaved and low-pressure-cured block are noted.

"Elastic Design of Prestressed Sections in Flexure by Charts or Tables" is the work of WITOLD W. ZAWILSKI, who gives attention to economies in both steel and concrete. A rapid design is demonstrated using charts or tables based on two fundamental principles: the full use of concrete stresses; and full use of the lever arm of the internal resisting couple. The writer also presents a design by charts and tables for composite cross sections, particularly as applied to bridge decks.

A wide range of aggregate gradings including extreme gap grading was tested by B. G. SINGH in studying the "Effect of the Specific Surface of Aggregates on the Consistency of Concrete." Marked effects of specific surface on consistency were measured. Aggregate gradings of the same specific surface have practically the same concrete-making properties. As the specific surface of the aggregate increases, consistency decreases, other factors being equal. However, attention must be given to tendencies toward segregation.

WALTER RILEY'S work on "Analysis of Continuous Arches on Flexible Piers" will appear in April. He demonstrates the applicability of the moment distribution method when combined with the principles of superposition to the practical analysis of continuous symmetrical arches on flexible piers.

Guide for Painting Concrete*

(For paints other than portland cement paint)

Reported by ACI Committee 616

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SYNOPSIS

This report describes types of paints, other than portland cement paint, commonly used on concrete. It reviews procedures for preparing the surface, selecting and applying the paint, and for repainting. Dampproofing and waterproofing are discussed briefly. A list of federal specifications for paints suitable for concrete is included.

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INTRODUCTION

This report considers primarily the decorative painting of portland cement concrete in buildings. The information can be applied generally to concrete of other structures where conditions are similar. Concrete may be either cast-in-place or precast units, and in this instance will include cement mortar, cement plaster, stucco, and asbestos cement products.

*Received by the Institute Dec. 3, 1956. Title No. 53-46 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 9, Mar. 1957, *Proceedings* V. 53. Separate prints in covers are available at 60 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than July 1, 1957. Address P.O. Box 4754, Redford Station, Detroit 19, Mich.

This report in form and substance as here submitted was approved unanimously by the committee as listed above.

†Deceased.

The use of clear coatings to impart water repellency is included. Dampproofing and waterproofing are discussed briefly with reference to bituminous coatings applied to the exterior surface of basement walls. Industrial applications of coatings to protect concrete against chemical attack are beyond the scope of this report.

There are hundreds of variations of paints marketed for use on concrete. Since it is obviously impossible to consider each of them in this report, discussion is based on general types. In good quality paints of the same general type, there will not be significant differences in performance resulting from minor variations in composition. The user's concern is to select a good quality paint of the proper type for a given condition and then to apply the paint in a manner conducive to obtaining best results. This report is intended to serve as a guide to these objectives.

Recommended practice for the use of portland cement paint on concrete is covered separately in ACI 616-49.* This type of paint, therefore, is not considered in this report.

TYPES OF PAINT AND APPROPRIATE USAGE

Concrete surfaces are often thought of as requiring special paints for satisfactory performance. Actually, most paints can be used effectively on concrete if proper attention is given to conditioning of the concrete and surface preparation.

TABLE 1—COMMON USAGE OF PAINT ON CONCRETE

General type of paint	Outside walls				Inside walls (partitions)	Floors		Surfaces exposed to water
	Above grade		Below grade			On subgrade	Not on subgrade	
	Exterior surface	Interior surface	Interior surface	Buried surface				
Oil-base	x							
Varnish-base		x			x		x	
Lacquer-base	x					x		x
Water-thinned	x	x	x		x			
Clear coatings	x							
Bituminous coatings				x				x

For the purpose of this report, paints are classified in six general types: oil-base, varnish-base, lacquer-base, water-thinned, clear coatings, and bituminous coatings. As will be noted, there are variations within general types. Also, there is overlapping between certain types. Table 1 indicates the more common usage of the general types of paint. The following para-

*"Recommended Practice for the Application of Portland Cement Paint to Concrete Surfaces (ACI 616-49)," ACI JOURNAL, Sept. 1949, *Proc.* V. 46, pp. 1-16.

graphs will aid in the selection of a specific type of paint for a particular usage.

Special priming paints are not required with most paints marketed for use on concrete. This is particularly true of lacquer-base paints and water-thinned paints. Some manufacturers provide a separate priming paint of increased alkali resistance for use with oil-base paints. Such a primer is of particular value if the concrete must be painted before it is fully seasoned (see section on conditioning of concrete for painting, p. 823). Special primers, or undercoaters, are also furnished for use with varnish-base paints. Generally, the label on the paint container will provide instructions concerning the need for a special primer or for altering the paint by thinning or adding a special ingredient for application as the first coat.

Oil-base paints

Oil-base paints consist principally of drying oil, more often linseed oil, and pigments. These are the conventional house paints commonly used on wood exposed to the weather. Sometimes a small proportion of resin or specially treated oil or varnish is added to "fortify" the paint. Stucco and masonry paints, which are marketed especially for exterior concrete surfaces, are oil-base paints that usually include a fortifying component to give added resistance to the alkali in concrete.

Oil-base paints are suitable for concrete walls exposed to the weather. They are glossy initially but are formulated to chalk gradually and in so doing to maintain a comparatively clean surface and to be in good condition for repainting when this becomes necessary. Tinted oil-base paints are formulated to chalk less than white paints in order to better retain their original color.

Straight oil paints perform well if the concrete is aged and dry. The fortified varieties are better if the concrete is subject to dampness. Neither will serve well if exposed for prolonged periods under water or where moisture will continue to reach the paint from the interior of the concrete. Both, however, will provide an effective barrier against penetration of rain water. Oil-base paints are not sufficiently abrasion resistant for use on floors. They may be used on dry inside walls, but usually faster and harder drying and better leveling paints are desired.

Varnish-base paints

Varnish-base paints consist principally of varnish, which is a heat-reacted mixture of drying oil and resin thinned with organic solvent, and pigments. Included, because of their similarity in properties are paints prepared with specially processed drying oils and solvent in place of varnish.

These paints are particularly suitable for dry interior walls. They create a smoother film and are faster drying than exterior oil paints and are marketed in gloss ranges from flat to semigloss and full gloss. Flat paints are commonly employed in living or office or working areas. Semigloss and gloss paints, often called enamels, are used where more frequent washing of the painted walls may be necessary. Varnish-base paints are prepared also as floor enamels and are suitable for dry concrete floors. If the floors are to be wetted fre-

quently or if moisture can accumulate within the concrete, as from contact with damp subgrade, lacquer-base paint is preferred (second footnote, p. 821).

Varnish-base paints, suitably formulated, have been employed satisfactorily on exterior surfaces. However, oil-base paints are more commonly used for this purpose.

Lacquer-base paints

Lacquer-base paints consist principally of synthetic resin dissolved in an organic solvent other than turpentine or mineral spirits, and pigments. Some may be modified by the addition of drying oil; frequently special plasticizing agents are used. These paints dry and harden quickly and completely, or essentially so, by evaporation of the solvent. Such drying is a characteristic of lacquers. Although lacquer may be more commonly thought of as a nitrocellulose or celloluse acetate product, the resin-based materials described here are, in effect, lacquers also.

Lacquer-base paints of the types manufactured for use on concrete have a high degree of resistance to the alkaline constituents of concrete. As a result, unlike oil-base paints and most varnish-base paints, they can be used where the concrete will be damp. Thus, paints prepared with a chlorinated rubber or a styrene-butadiene resin of high styrene content are suitable for concrete floors resting on subgrade. Vinyl-resin paints are used mostly on walls. All can be used effectively where the concrete will be wet, as in shower stalls, wash rooms, laundry rooms, and on weather-exposed concrete.

Epoxy-resin paints are relatively new, but appear to be well suited to concrete painting. The straight epoxy lacquer requires the addition of a catalyst just before application to accomplish proper hardening of the film. The "modified" epoxy paints, i.e., containing some drying oil, do not require a catalyst and dry more slowly than the full lacquer type. Epoxy-resin paints are alkali-resistant and are said to have excellent durability in both interior and exterior exposure. Both types are marketed for use on concrete floors. The unmodified epoxy paint, especially, has excellent resistance to abrasion.

The lacquer-base paints, in general, contain stronger solvents and are not as agreeable and easy to work with as oil- and varnish-base paints. Therefore, they are not usually used where oil- or varnish-base paint will suffice.

Water-thinned paints

Water-thinned paints are typified by the use of water in place of solvent for liquefying the paint. Otherwise, most water-thinned paints are similar to the others, containing resins or oils or both as binder, and pigments. An immediate advantage of water-thinned paint is the absence of solvent fumes, which is especially appreciated in confined working areas. Another advantage is that application equipment is easily cleaned with water. A disadvantage in some circumstances is that water-thinned paints may be damaged in their containers by freezing. Until comparatively recently, these paints were considerably less durable, particularly lacking in scrubbing resistance, than

solvent-thinned paints. Today, however, there are many durable water-thinned paints that are suitable for concrete in certain exposures.

Calcimine is one of the oldest water-thinned paints. It lacks the durability of most other paints and is suggested for use only on dry walls where a cheap or temporary paint is desired. Casein and other protein base and oleoresinous base water-thinned paints are suitable for interior dry concrete, and some alkyd resin drying oil formulations have been used effectively on exterior concrete.

These earlier types have given way to a large extent to the now popular latex paints, often called "rubber-base" paints.* The first of the latex paints, and still the most common, was made with a styrene-butadiene latex, similar to but not identical with synthetic GR-S rubber. More recently, paints based on polyvinyl acetate, acrylic, and other resin latexes, have been marketed. Latex paints are particularly suited to the decoration of interior concrete walls, and are probably the best for painting the interior surface of below-grade basement walls, because they can be formulated to be resistant to alkali and to "breathe," thus minimizing hydrostatic pressure build-up behind the film.†

The use of latex paint on exterior concrete has been slower to develop than its use on interior concrete. However, best evidence today indicates that properly formulated latex paints will have good exterior durability. The user is cautioned to make sure the paint is manufactured expressly for exterior use.

Latex paints, after thorough drying, have excellent washability. They should not be used on floors because of insufficient resistance to abrasion.

Clear coatings

Clear coatings, although not paints in the usual sense of the word, are included because they are sometimes applied to exterior concrete walls as a water repellent, where it is not desired to change appearance. Waxes, resins, soaps (stearates), and drying oils, often tung oil, dissolved in a petroleum solvent, are commonly used for this purpose. Water-thinned varieties are also available. Recently silicone resins are being added, or used singly, to increase the water repellency of such products.

Clear coatings are generally effective in minimizing penetration of moisture, as from rain, into concrete. They are not recommended where appreciable hydrostatic pressure may develop, as on below-grade concrete walls, either for the inside or the outside surface. Clear films are not as durable as most paint films, because they lack pigments which tend to protect against the destructive action of sunlight.

Clear coatings are also used to prevent dusting of concrete floors. Usually, inorganic compounds, such as silicates and fluosilicates, dissolved in water, are employed for this purpose.

*Water-thinned rubber base paints, which are more aptly described as latex paints, should not be confused with solvent-thinned lacquer-base types, which are sometimes designated as rubber-base paints.

†The use of any paint where appreciable hydrostatic pressure may develop behind the film is questionable. Even though chemically resistant to alkaline water, the paint may blister and subsequently peel. See also ACI 616-49, "Recommended Practice for the Application of Portland Cement Paint to Concrete Surfaces."

Bituminous coatings

Bituminous coatings are included because of their wide use in dampproofing and waterproofing concrete, such as basement walls, where appearance is not a factor. These are black coal-tar or asphalt products, which are furnished as solids to be melted for hot application, or as liquids, i.e., solids thinned with solvent or emulsified with water, for application at ordinary temperatures. The liquid materials, applied in comparatively thin films, are satisfactory for dampproofing, but hot applications of greater thickness are desirable for waterproofing, i.e., where considerable hydrostatic pressure will be exerted upon the coating. Better waterproofing effect on concrete is obtained by building up layers of the hot bituminous material and reinforcing fabrics or felts. The built-up membrane type of coating has a distinct advantage if cracks develop in the concrete, since it is better able than a comparatively thin, unreinforced coating to maintain continuity over the crack.

In this discussion, consideration has been given only to the application of dampproofing and waterproofing material to the concrete surface exposed to the soil or water. Lasting effects cannot usually be expected if the coating is applied so that pressure will be developed behind it, as it may on the interior surface of a basement wall.

SELECTING THE PAINT

After establishing the type of paint he will use, the user must still select a particular product. Many of the larger organizations, including government agencies, procure paint by seeking bids under a controlling specification. This procedure is effective provided the specification is adequate and the purchaser can be sure that the paint conforms with the specification. A list of federal specifications for various paints used on concrete and brief notes concerning their composition and use will be found at the end of this report.

If a specification is not employed, the user should try to find a trade-name product known to have performed satisfactorily under similar conditions. He should be careful to assure himself that the information he obtains is unbiased and accurate. Experienced representatives of paint manufacturers of good standing in the community, or nationally, may be relied upon to give sound advice, because they have much at stake in the good performance of their product. Others may lack sufficient knowledge to advise correctly or may be willing to risk poor performance in the immediate interests of obtaining the order.

It is customary among reliable manufacturers to state the proper use and the general composition of the product on the label of the container. The label composition data will indicate the general type, but not usually the specific resin or other binder ingredients. Sometimes the salesman can fill in this information or obtain it from the home office. In any event, the label will often furnish valuable guidance in selecting a suitable product.

Cost of the paint is secondary to making sure it is the proper type and of

good quality. Paint cost represents only 15 to 25 percent of the total expenditure for the paint job, and a dollar or two saved on a gallon of paint will prove to be false economy if early failure is experienced because of poor quality of materials. On the other hand, high paint cost does not necessarily insure quality. In general, paints of the same type marketed by reliable manufacturers will be reasonably close in price.

CONDITIONING CONCRETE FOR PAINTING

With few exceptions, paint, including clear coatings, will serve best if the concrete is seasoned and dry when painted. When new, concrete is damp in the interior even though surface dry and this impounded moisture may later cause blistering and peeling of the paint film. Also, new concrete has a high degree of alkalinity, which is particularly harmful to paints containing drying oils because of their susceptibility to saponification. Although lacquer-base paints and latexes are generally resistant to alkalies, dampness may cause blistering.

Surface treatments with acids or acid salts such as zinc sulfate to reduce alkalinity of new concrete have been tried with varied results. Such treatments at best provide only temporary neutrality to the surface; they do not affect the alkaline reserve within the concrete and any later outward movement of moisture will bring alkalies into contact with the paint film. Good results in laboratory tests were reported for a neutralizing treatment consisting of 3 percent phosphoric acid and 2 percent zinc chloride,[7] and if concrete must be painted when new, this treatment is recommended. The solution is brushed or sprayed on the surface and allowed to dry 24 hr, following which residues formed in the reaction are brushed off before painting.

Seasoning is considered to be more reliable than surface treatment as a means of conditioning concrete for painting. With age, the surface alkalinity of concrete is reduced through carbonation. Also, the free moisture content is decreased, except, of course, where the concrete is exposed to a source of water. As a guide, it is recommended that concrete be at least 6 months old before painting with paints containing drying oils, and at least 2 months old before painting with alkali-resistant paints.

Seasoning or treatment of the concrete is generally unnecessary for bituminous coatings applied to concrete surfaces against which backfill will be placed. These coatings are alkali-resistant and because water or backfill pressure is developed against the coated surface there is little danger of blistering. Blistering can occur, however, if the coating is applied while the concrete is still moist and the coated surface is then exposed to direct sunlight. If exposure to sunlight is unavoidable, the application of whitewash over the bituminous coating will minimize blistering until backfill is placed or the coated surface is otherwise covered. Concrete in any case should be surface dry to obtain good adhesion of hot-applied or solvent-thinned bituminous coatings; a damp surface is permissible for a water-thinned coating.

SURFACE PREPARATION

Paint cannot be expected to adhere well to concrete if interfering substances are on the surface. Thus, any dirt, dust, grease, oil, and efflorescence should be removed before painting. Dirt and dust may be removed by air-blowing, brushing, hosing, or scrubbing, depending upon the character and amount of the contamination. For extremely dirty surfaces, blast cleaning may be found desirable. Grease and oil may be washed off with solvent or strongly alkaline solutions such as lye. Efflorescence may be removed by washing with dilute (1 to 4) muriatic acid or by blast cleaning. Thorough rinsing of the surface with water should follow acid or lye treatment.

Efflorescence is the result of deposition of salts from water moving from the interior of concrete and evaporating on the surface. If water continues to enter concrete after painting, efflorescence likely will develop anew and disrupt the paint film. Therefore, in removing efflorescence, consideration should be given to this possibility and the source of moisture eliminated if possible.

Extremely dense, glazed surfaces should be roughened slightly by light sandblasting, rubbing with abrasive, or etching with acid. Etching of steel-trowelled floors with 1 to 4 muriatic acid solution, followed by rinsing, is common practice. The phosphoric acid-zinc chloride pretreatment solution, described on p. 823, is preferable if the concrete is new.

In some cases, with concrete block or other rough surfaces, it may be desired to smooth out the surface or fill in small pits and cracks of concrete before painting. A 1:1 cement-sand grout using a mortar sand is recommended for this purpose. The surface should first be well wetted, and the grout scrubbed in with a stiff brush. A 1:2 or 1:3 cement-sand mortar should be used to fill large cracks and holes. Loose or defective concrete and dirt should be removed and the area dampened before filling with mortar. The mortar should be plastic, but no wetter than necessary, and should be packed firmly in place. Curing for 2 or 3 days, as by frequent spraying with water or covering with wet burlap, should be provided after the grout or mortar has set. As for new concrete, painting, except with bituminous coatings, should be delayed to permit the patchwork to dry. Treatment with the phosphoric acid-zinc chloride solution is recommended if painting cannot be delayed.

PREPARATION OF PAINT

Paints must be thoroughly stirred or agitated just before use to create a uniform distribution of pigment throughout the liquid. Hand stirring should be done with a broad flat paddle. Power stirrers and shakers are commercially available and their use will greatly minimize labor and shorten the time needed for complete mixing.

Most paint containers are filled close to the top and it is difficult to stir the paint without slopping. For this reason, it is good practice to pour off a portion of the paint into a separate clean container, vigorously stir the

remainder and then recombine the two, stirring slowly. Often, painters will finish off the mixing by pouring the paint back and forth between the two containers. With a shaker, the paint is mixed completely within its original container.

Clear coatings and liquid bituminous coatings, even though not containing pigments, should be stirred or agitated to assure homogeneity of the materials.

Any thinning of paint should be done only at the manufacturer's direction and with the recommended thinner. Thinning instructions will usually appear on the label of the container. Indiscriminate thinning is a constant hazard to successful painting. Common thinning errors include using turpentine or mineral spirits with lacquer-base paints and organic solvent of any type with water-thinned paints. Lacquer-base paints generally require special thinners, and water-thinned paints should be thinned only with water.

Tinting also should be done with discretion. The usual colors-in-oil are made primarily for use with oil-base and varnish-base paints, and may not be suitable for lacquer-base paint. Special tinting pastes are generally required for water-thinned paints. Some manufacturers furnish a universal tinting paste that can be used with two or more types of paint. The paint manufacturer's instructions in regard to tinting should be followed.

Some paints require special preparation. Catalyzed epoxy-resin paint, for example, requires the addition of the catalyst just before use and thereafter will have a limited pot life. Aluminum paint may be furnished with pigment and vehicle in separate containers, requiring mixing just before use. Melting and heating of solid bituminous materials are, of course, required for their application. The manufacturer furnishes complete directions for such preparations.

APPLICATION OF PAINT

Painting, like most construction work, is best done under favorable climatic conditions. Paint increases in viscosity with cold; it becomes more difficult to apply and does not wet the surface as well as when warm. Even though the paint is warmed, relatively poor paint work will result if the concrete is cold. It is best to paint when the temperature of the air and concrete is above 50 F. Many consider 45 F as the minimum for a satisfactory job.

The importance of painting concrete when dry is discussed under the section on conditioning concrete for painting, p. 823. Accordingly, painting when rain may strike the surface should be avoided; the same is true when condensed moisture is on the surface. An exception exists in water-thinned paints, which may be safely applied to damp, although not wet, surfaces.

Method of application

The method of applying paint is not usually too important, provided the work is well executed. Brush, spray, and roller coating are all good methods. However, if the paint manufacturer recommends a specific method, it is best to use it. Some lacquer-base paints, e.g., the vinyls, are fast drying and,

therefore, are better adapted to spraying than to brushing. If brushed, other than for the first coat, their strong solvent action may disrupt the underlying coat. Some prefer to have the first coat brushed regardless of how succeeding coats are applied. They believe that brushing will work the paint better into surface irregularities and that surface dust will be less apt to interfere with adhesion of the paint. There is no doubt that this is a highly satisfactory procedure, but spray application of all coats can also be successful. Roller coating came into prominence with the development of latex paints and is now commonly employed for these and other paints, particularly where applied to interior walls.

Good workmanship is important to obtaining good appearance and serviceability of the paint film. With any method, paint should be applied as a full, wet film, but not, of course, so heavily that it will run or sag. Care must be taken to see that paint is worked into surface depressions; that the coverage is uniform; and that good blending of the film is obtained at laps. In spraying, air pressures should be just sufficient for effective atomization of the paint. Excessive pressures will result in unnecessary loss of paint and may have adverse effects on the paint film.

Number of coats and thickness

The number of coats and thickness of the completed coating are important considerations. It is seldom that one coat of any type of paint will provide the desired appearance and serviceability. For good durabilty in exterior exposure, a complete, dried coating should have a thickness of about, but at least, 0.005 in. This thickness will be achieved with two coats of oil-base paint if applied at about 500 sq ft per gal. each on a relatively smooth surface. Some prefer to apply three coats, in which case the paint may be spread more thinly in each coating. If the surface is rough, the paint will have to be spread at a lower over-all rate, down to 250 sq ft per gal., to obtain the desired film thickness. Lacquer-base paints, generally, contain a lesser proportion of nonvolatile solids than oil-base paints, and will require three or four coats at 500 sq ft per gal. each to provide the desired dry film thickness.

For interior decorative painting, thickness of coating as such is of lesser importance. Two coats of paint are usually needed to obtain good appearance whether the paint is varnish-base, lacquer-base, or water-thinned. If the coating will be subject to considerable washing, three coats will be desirable. Concrete floors are usually given two coats, brushed out to about 600 sq ft per gal. each.

When more than one coat of the same paint is to be applied, it is good practice to apply alternate coats in slightly contrasting shades to distinguish readily between coats. This can be done with tinting colors or by adding a small amount of differently colored paint of the same type.

Drying time

The drying time required before an additional coat of paint is applied will vary over a wide range depending upon the type of paint and atmospheric

conditions. Straight oil-base paints are the slowest drying, requiring at least 24 hr, preferably 48 hr, even in warm weather. The fortified oil-base paints and varnish-base paints dry faster; overnight drying is usually sufficient for these. Lacquer-base paints and water-thinned paints are fast drying, requiring from as little as 30 min to a few hours to be ready for recoating. In cool, humid atmospheres the drying time of all paints should be extended. The paint manufacturer will advise on proper drying time for his particular product.

Clear coatings

Clear coatings are commonly applied by brush or spray, in one or two coats. These materials are quite thin in consistency and the first coat will penetrate to some extent into most concrete surfaces. This is desirable from the standpoint of retaining dampproofing effectiveness, because surface films will weather off rather quickly. The second coat may be desired to compensate for excessive penetration of the first coat or to create a surface film for brighter appearance. Depending upon the viscosity of the coating material and the porosity and roughness of the concrete, coverage for the first coat can vary from about 100 to 500 sq ft per gal. The second coat will cover from about 400 to 800 sq ft per gal. Drying time required between coats will depend upon basic composition as discussed above.

Bituminous coatings

Bituminous coatings are applied more thickly than decorative paints. The thinned materials are applied by brushing or spraying in one or more coats, depending on the desired degree of dampproofing effect. Usually two coats are desirable, if for no other reason than that the second coat will tend to seal imperfections in the first. If the material is quite viscous, it should be thinned for the first coat to provide for more effective penetration and adhesion. Each coat should be applied as thickly as possible without incurring excessive runs and sags. The coverage can be expected to be from around 100 to 300 sq ft per gal. per coat.

Bituminous materials requiring melting for application should be applied heavily, about 1/16 in. thick for each coat. A single coat is often adequate for basement walls. However, if the waterproofing will be exposed to severe conditions, a built-up membrane coating, alternating hot material with reinforcing fabric or felt, is desirable. The reinforcement should overlap and be bonded to the hot material. A thin prime coat of a liquid bituminous material, either solvent or water-thinned, is sometimes applied to improve adhesion of the hot application; if the concrete is clean and dry, the hot material may be applied directly. Application of the hot coat is made by brushing, mopping, or spraying.

REPAINTING

Where it is desired to repaint with the same type of paint as presently on the surface, and the old paint is in good condition, the procedure is simply

a matter of cleaning and applying a single coat of paint. Cleaning methods discussed under section on surface preparation (p. 824) for new concrete are generally applicable. Washing with detergents before painting may be necessary if dirt accumulation is excessive and tenacious, such as to resist ordinary dusting or hosing.

If the old paint has become partially removed, but that remaining is in good condition, the bare areas should be painted first, followed by a coat over all. An additional coat may be necessary for best appearance, especially if a pronounced change in color is involved.

If the old paint is in generally poor condition, as from blistering, peeling, or disruption from efflorescence, consideration should first be given to the cause. The concrete may have been painted while too new, or a source of supply of moisture to the concrete may still be present. If the latter is true, any new paint will probably also be unsatisfactory unless the source of moisture can be eliminated. Consideration should also be given to changing the type of paint (see section on types of paint and appropriate usage, p. 818.)

In the majority of cases, paint of one type can be applied satisfactorily over paint of another type. However, the paint manufacturer's advice should be sought to make sure in any particular case. Lacquer-base paints are apt to give trouble if applied over other types because of their strong solvent action, e.g., a rubber-base floor paint applied over a varnish-base floor paint. Loose paint, curled edges, and blistered paint should be removed in the cleaning operation before painting. It may be preferable to remove all of the old paint. Light sandblasting, machine sanding, or paint-and-varnish remover may be utilized as most expedient. Filling of any cracks and holes in the concrete should be done in the same manner as for new concrete (see section on surface preparation, p. 824).

SPECIFICATIONS FOR PAINTS FOR CONCRETE

Following are listed federal specifications for various paints considered suitable for use on concrete. The designation in force at the time of writing is indicated. Later designations should be used as they become available. Copies of federal specifications are available at regional offices of the General Services Administration, or may be purchased from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. The General Services Administration offices are located in Atlanta, Boston, Chicago, Dallas, Denver, Kansas City, New York City, San Francisco, Seattle, and Washington, D. C.

Oil-base paints

TT-P-24 Paint; concrete and masonry, exterior, egg-shell finish, ready-mixed, white and tints

A "fortified" paint is contemplated under this specification. White lead, zinc oxide, and titanium dioxide are the basic white pigments. Type I should be specified for white paint only, Type II for tinted paint. Type II is formulated to chalk less to minimize fading of color. For the first (prime) coat, this paint should be modified by adding spar

varnish and thinner at the rate of 2 qt and 1 qt, respectively, to a gallon of the paint. Spar varnish may be procured under Federal Specification TT-V-121c and thinner under TT-T-291a, Grade I.

TT-P-25a Primer; paint exterior, undercoat for wood, ready-mixed, white and light tints

Although described as a primer for wood, this paint is also suitable as the first coat on concrete where a straight oil-base finish paint will be used. It should never be used as a top coat. Finish paint, such as furnished under TT-P-102 and TT-P-103, should normally be applied within 2 weeks after application of this priming paint. Although usually white, this paint may be obtained in light tints. It is based on white lead and titanium dioxide pigments.

TT-P-81c Paint; ready-mixed, exterior, medium shades on a lead-zinc base

Medium shade paints in gray, green, and blue may be procured under this specification. Tinted or light shade paints should be procured under TT-P-102, or TT-P-104. This paint incorporates white lead and zinc oxide as basic white pigments and is fortified with alkyd resin. TT-P-25a is a suitable priming paint.

TT-P-102 Paint; titanium-lead-zinc and oil, exterior, ready-mixed, white and light tints: Class A—for white; Class B—for light tints

This is a typical house paint based on white lead, zinc oxide, and titanium dioxide pigments. Class A which contains a chalking type of titanium dioxide should be used for white. Class B which contains nonchalking titanium dioxide should be used for light tints. TT-P-25a is recommended for the prime coat.

TT-P-103 Paint; titanium-zinc and oil, exterior, fume-resistant, ready-mixed, white

This house paint is furnished in white only. It contains zinc oxide and titanium dioxide pigments, but no lead pigments. The lead pigments are purposely omitted to provide a paint that will not blacken in atmospheres containing hydrogen sulfide. It is not recommended for use otherwise. The paint is not to be tinted because tinting colors also will darken in hydrogen sulfide. For the first coat, this paint should be thinned with 1 pint of linseed oil to the gallon.

TT-P-104 Paint; white lead and oil, exterior, ready-mixed, white and light tints

This is a pure white lead house paint. For the first coat, the paint should be thinned with 1 pint of linseed oil to the gallon.

Varnish-base paints

TT-P-0030 Paint; alkyd, interior, flat, white and light tints

Oil-modified alkyd resin paints such as described by this interim specification are considered high quality paints for interior decoration. The paint may be applied directly to concrete or over primer-sealer TT-P-56b.

TT-P-47a Paint; oil, interior, nonpenetrating flat, ready-mixed, tints and white

Although this paint is a "one coat" type, two coats may be necessary to obtain the desired appearance. It is contemplated that the paint will be based on processed drying oils and a small amount of resin, i.e., a long-oil varnish. It is intended to be applied directly to the concrete, without the need for a priming coat.

TT-P-51d Paint; oil, interior, flat, white and tints

This paint is based on processed drying oils or varnish or a mixture thereof. It is intended to be applied over primer-sealer TT-P-56b.

TT-P-56b Primer coating (primer-sealer); pigmented oil, plaster and wall board

This priming paint is intended for use with various interior finish paints, such as TT-P-51d, TT-E-506c, and TT-E-508. It is of particular value if the concrete surface tends to be porous. It is no more alkali resistant than other oil- or varnish-base paints. It is available in white or light tints.

TT-E-506c Enamel; tints and white, gloss, interior

This is a high-gloss, oil-modified alkyd resin paint having excellent washability. Enamel undercoat TT-E-543 is recommended for the first coat unless the surface is

porous in which case primer-sealer TT-P-56b should be used as the first coat and TT-E-543 as the second coat.

TT-E-508 Enamel; interior, semigloss, tints and white

This is a semigloss, oil-modified alkyd resin paint having good washability. Prime painting should be as described for TT-E-506c, or TT-E-508 may be applied directly over TT-P-56b.

TT-E-487 Enamel; floor and deck

A tough and flexible varnish is required in this paint, which can be procured in any dark or medium shade. For the first coat it is suggested that 1 qt of a mixture containing 2/3 spar varnish and 1/3 turpentine or mineral spirits be added to a gallon of the paint.

TT-E-543 Enamel undercoat; interior, tints and white

This paint is intended for use as the first coat under gloss and semigloss enamels, TT-E-506c and TT-E-508. If the surface is porous it should first be sealed with primer-sealer TT-P-56b.

Lacquer-base paints

TT-P-91a Paint; rubber-base, for concrete floors

The rubber base of this paint may be either natural or synthetic. The paint is available in various shades of gray, tile red, brown, green, and black. It is not intended for exterior use. It may be used where appropriate on interior walls, but its principal use is on concrete basement floors. The paint is intended to be applied as furnished for all coats.

TT-P-95 Paint; rubber-base, for swimming pools

Although primarily intended for swimming pools, this rubber-base paint is also suitable for shower rooms and other surfaces, either interior or exterior, exposed to water. It is usually furnished in green, blue, or white. For the first coat, the paint should be thinned with 1 qt of xylol (TT-X-916) to the gallon.

Water-thinned paints

TT-C-96 Calcimine; cold- and hot-water types

This is a temporary type of paint that can be readily washed off with warm water. It is furnished as a powder in white or light tints. Type I powder is mixed with cold water, Type II with boiling water. The latter is considered to have smoother brushing properties. It may be applied directly to concrete.

TT-P-18 Paint; alkyd resin-emulsion exterior, paste, tints and white

This paint is furnished as a paste to be diluted with about ½ gal. of water to a gallon of paste to obtain a consistency slightly thinner than an oil paint. It is intended to be applied directly to concrete without a special prime coat. The white pigment of this paint is required to be titanium dioxide.

TT-P-22 Paint; cold-water, exterior, powder (with mixing liquid)

This is a casein paint of limited service life. Another paint should be used where durability is required. It is furnished as a powder in white or light tints with a mixing liquid, usually boiled linseed oil. Paint is prepared on the job by mixing 90-100 lb of powder to 10-12 gal. of water and 1 gal. of mixing liquid. It is applied directly to concrete, which should be damp when the paint is applied.

TT-P-23a Paint; cold-water, interior, light tints and white

This casein (or other protein) paint is furnished either as a powder, Type I, or as a paste, Type II. The powder is mixed with water at the rate of 8-10 lb to a gallon. The paste is mixed with ½ its volume of water. It is applied directly to concrete, or over primer-sealer TT-P-56b. This interior paint has but moderate resistance to washing.

TT-P-29 Paint; latex-base, interior, flat, white and tints

Paint procured under this specification has good washability. Type I is furnished at painting consistency. Type II is a paste to be thinned with 1 pint of water to a gallon of paste. It is applied directly to concrete, or over a flat primer such as TT-P-56b. If used for repainting a glossy surface, the surface should first be dulled as by sanding. Note that this latex paint is intended only for interior exposure. A federal specification for an exterior latex paint has not as yet been issued.

TT-P-88a Paint; resin-base emulsion, interior, paste, white and tints

It is contemplated that the binder of this paint will be oleoresinous. The paint has better washability than casein paints but not as good as the latex paint described by TT-P-29. The paint is furnished as a paste to be reduced to painting consistency with 1 volume of water to 2 volumes of paste. It may be applied directly to concrete or over a primer such as TT-P-56b.

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Title No. 53-47

Final report of an investigation sponsored by the Reinforced Concrete Research Council at the University of Illinois

Shear Strength of Reinforced Concrete Frame Members Without Web Reinforcement*

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SYNOPSIS

Experimental and analytical studies reported in this paper have shown conclusively that the strength of a reinforced concrete frame member without web reinforcement is affected by shear through the formation of diagonal tension cracks. If the percentage of longitudinal reinforcement is small or the shear span is very long, a member without web reinforcement fails in flexure without prior formation of diagonal tension cracks; in such case, strength of the member is unaffected by shear. If, however, a diagonal tension crack forms, the strength of the beam is usually lower than that corresponding to flexural failure. Depending on its make-up, such member may fail either simultaneously with the formation of the diagonal tension crack or it may fail at a higher load. The first type of failure is designated as diagonal tension failure; the second is designated as shear compression failure. The relationship between flexural, diagonal tension, and shear compression failures is discussed in detail in this paper.

The tests included 33 knee frames subjected to combination of axial compression, shear, and moment, and 38 stub beams subjected to moment and shear. In all knee frames the axial load was equal to the external shear. The major variables were the length of shear span, the strength of concrete, and the percentage of reinforcement.

Analytical expressions are presented for predicting the diagonal tension cracking load and shear compression strength. The diagonal tension cracking load is expressed in terms of the nominal shearing stress, and the shear compression strength is expressed in terms of the shear moment capacity.

INTRODUCTION

Reinforced concrete frame members are usually subjected to a combination of bending, axial force, and shear, but in the design for shear the axial force is neglected. Recognizing the complete absence of experimental evidence on effects of axial force on the shear strength of reinforced concrete structural members, the joint ACI-ASCE committee on shear and diagonal tension outlined an investigation of frame members without web reinforcement. The investigation was carried out at the University of Illinois under the sponsorship of the Reinforced Concrete Research Council in 1953-1955.

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The primary purpose of the investigation was to determine the effect of axial *compression* on the shear strength of reinforced concrete members without web reinforcement. The secondary objective was to obtain some data on how the shear strength is influenced by the formation of plastic hinges. Such information should make it possible to determine under what conditions reinforced concrete members may be designed without web reinforcement, and such information may be utilized further in studies of the effects of web reinforcement.

Recent studies of simple and restrained reinforced concrete beams[1,2,3] resulted in formulation of new concepts regarding the effects of shear on strength of reinforced concrete. This investigation offered a further opportunity for checking and extending the new concepts.

DEFINITIONS AND NOTATIONS

Critical section is the section of potential shear compression failure. In a knee frame, it is the cross section passing through the knee corner; in a stub beam it is the cross section at the face of the column stub.

Diagonal tension crack is an inclined crack, extending from the longitudinal reinforcement to the critical section and making an angle of approximately 45 deg with the longitudinal reinforcement at the level of the steel.

Diagonal tension cracking load is the load at which a diagonal tension crack can be first clearly observed.

Diagonal tension failure is a failure simultaneous with the formation of the first diagonal tension crack.

Shear compression failure is caused by destruction of the compression zone of concrete at the critical section above an existing diagonal tension crack.

a = length of the shear span (Fig. 1)

a_c = distance from the section of zero moment (with respect to the centroidal axis) to the starting point of diagonal tension crack

b = width of cross section

C = compressive force in concrete (Fig. 11)

d = effective depth of reinforcement

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

f_c' = compressive strength of 6 x 12-in. concrete control cylinders

f_r' = modulus of rupture of concrete

f_s = steel stress at the critical section at shear failure

f_y = yield point stress of reinforcement

h = over-all depth of cross section

$j_u = 1 - \dfrac{k_2}{k_1 k_3} \dfrac{\alpha p f_s}{f_c'}$ (Fig. 11)

k_1 = coefficient defining the magnitude of the compressive force C at failure (Fig. 11)

k_2 = coefficient defining the position of the compressive force C in concrete (Fig. 11)

k_3 = ratio of the compressive strength of concrete in flexure to f_c' (Fig. 11)

K = strain reduction coefficient (Fig. 11)

M_{st} = moment around the centroid of tension reinforcement at the critical section

$(M/V)_c$ = ratio of the moment around the centroid of compressive force to the shear at the section of diagonal tension cracking

n = modular ratio

N = axial force

p = $\dfrac{\text{area of tension reinforcement}}{bd}$

P = load applied to the specimen

P_c = load at first diagonal tension cracking

P_y = load at first yielding of the reinforcement

P_u = failure load

T = tension force in longitudinal reinforcement (Fig. 11)

V = shear

v_c = $\dfrac{8V_c}{7bd}$, nominal shearing stress at diagonal tension cracking

v_{allow} = allowable shearing stress, ACI 318-56

v_u = $\dfrac{8V_u}{7bd}$, nominal shearing stress at failure

y = deflection of the end of the shear span in the direction perpendicular to the axis of the knee leg

α = $1 + \dfrac{N}{pbdf_s}$

ϵ_s = steel strain at shear failure (Fig. 11)

ϵ_u = maximum concrete strain at shear failure (Fig. 11)

SPECIMENS AND TESTS

Test program and specimens

An outline of the test program is given in Table 1. The investigation included two types of specimens: 33 knee frames and 38 stub beams. All specimens were of the same rectangular cross section and were reinforced in tension only. The major variables were the length of the shear span, the strength of concrete, and the percentage of reinforcement.

A typical knee frame is shown in Fig. 1a. All knee frames consisted of two equally long legs joined in a 90-deg angle at the knee. The end of each leg was formed into a loading stub fitted with a roller bearing block. Frames were loaded in a standing position along the load line shown in Fig. 1a.

A typical stub beam is shown in Fig. 1b. All beams had a reinforced column stub cast integrally at midspan, and one or two external stirrups placed at each end outside the span to prevent failure by splitting at the level of reinforcement. The beams were simply supported and loaded at midspan through the top of the column stub.

The dimensions of all specimens are listed in Table 2 and Fig. 1. Both the knee frames and the stub beams were approximately 12 x 16 in. in cross section, reinforced with 0.34–3.83 percent tension steel. The nominal strength of concrete was 2000, 4000, or 6000 psi, and the length of the shear span, a, was varied from 14 to 113 in. The effective depth of the reinforcement was approximately 14.5 in. The bars were placed in one layer except in specimens F38A6, F55A6, B14A6, B21A6, B56A6, and B70A6; in these, five bars were placed in a lower layer and the remaining bars in an upper layer with 1 in. clear spacing between layers. The bars were spaced evenly and symmetrically across the section.

The maximum bending stresses occurred in the knee frames at sections passing through the knee corner perpendicular to the shear spans, and in the stub beams at sections located at the faces of the column stub perpendicular to the beam axis. These sections will be referred to as critical sections.

Materials

Concrete was made with a standard brand of Type I portland cement, Wabash river gravel, and sand. The cement was purchased in six lots, the aggregates in eight lots. The average grading of the aggregates is shown in Table 3. The maximum size of the gravel was 1 in. and the fineness modulus of the sand was 2.87. The specific gravities of the gravel and the sand were 2.70 and 2.65, respectively, and the absorption of both aggregates was approximately 1 percent by weight.

Mixing was performed in a non-tilting horizontal drum mixer for 3 to 5 min. Materials were batched by weight in the following proportions: the cement to water ratios were 1.0,

1.5, and 2.0, and the cement to sand to gravel ratios were 1:5.25:7.25, 1:3.35:4.65, and 1:2.30:3.20 for the 2000, 4000, and 6000 psi mixes, respectively. The slump varied from 1 to 3 in. The average compressive strengths of concrete determined from control cylinders are listed in Table 4.

The reinforcement was made of hard grade deformed bars purchased in three lots. One tensile coupon was tested from each bar and the bars for every beam matched accordingly. The average yield points and ultimate strengths weighted for all bars of one specimen according to actual areas are listed in Table 4. The differences between the values for individual bars were small; furthermore, except for B21B6, in all specimens either none or all bars yielded at failure. The actual areas and other dimensions of the bars are listed in Table 5. The average modulus of elasticity for all bars was 28×10^6 psi.

Fabrication of specimens

The knee frames were cast in a horizontal position in a form made of a 16-in. steel base and 12 in. high plastic coated plywood sides. The reinforcement was formed into a rigid mat by welding transverse spacer bars at the ends and at the knee (Fig. 1a). The mat was then

TABLE 1—OUTLINE OF TEST PROGRAM*

Shear span, in.	Nominal strength of concrete, psi	Nominal percentage of reinforcement						
		A	B	C	D	E	F	G
Knee frames								
21.6	2000		1.89†			0.36		
	4000		1.89†	1.62†	1.25	0.86	0.76	0.36
	6000	2.40	1.89					
38.6	2000		1.89			0.51		
	4000		1.89		1.39	0.94		
	6000	2.86	1.89					
55.6	2000		1.89			0.54		
	4000	2.05	1.89		1.55	1.01		
	6000	3.20	1.89					
70.0	2000		1.89					
	4000	2.15						
	6000	3.45						
84.0	4000		1.89					
113.0	4000		1.89					
Stub beams								
14.0	2000		1.89			0.63		
	4000	2.51	1.89			1.26		
	6000	3.78	1.89					
21.0	2000		1.89			0.63		
	4000	2.51	1.89			1.26†	1.15	0.63
	6000	3.78	1.89					
28.0	2000		1.89			0.63		
	4000	2.51	1.89			1.26		
	6000	3.78	1.89					
40.0	4000		1.89					
56.0	2000		1.89			0.63		
	4000	2.51	1.89			1.26		
	6000	3.78	1.89					
70.0	2000		1.89					
	4000	2.51						
	6000	3.78						
84.0	4000		1.89					
113.0	4000		1.89†					

*All specimens were approximately 12 x 16 in. in cross section, reinforced in tension only. Effective depth was approximately 14.5 in. Reinforcement was of hard-grade billet steel. One specimen of each kind was tested except for types marked with dagger, of which two specimens were tested.

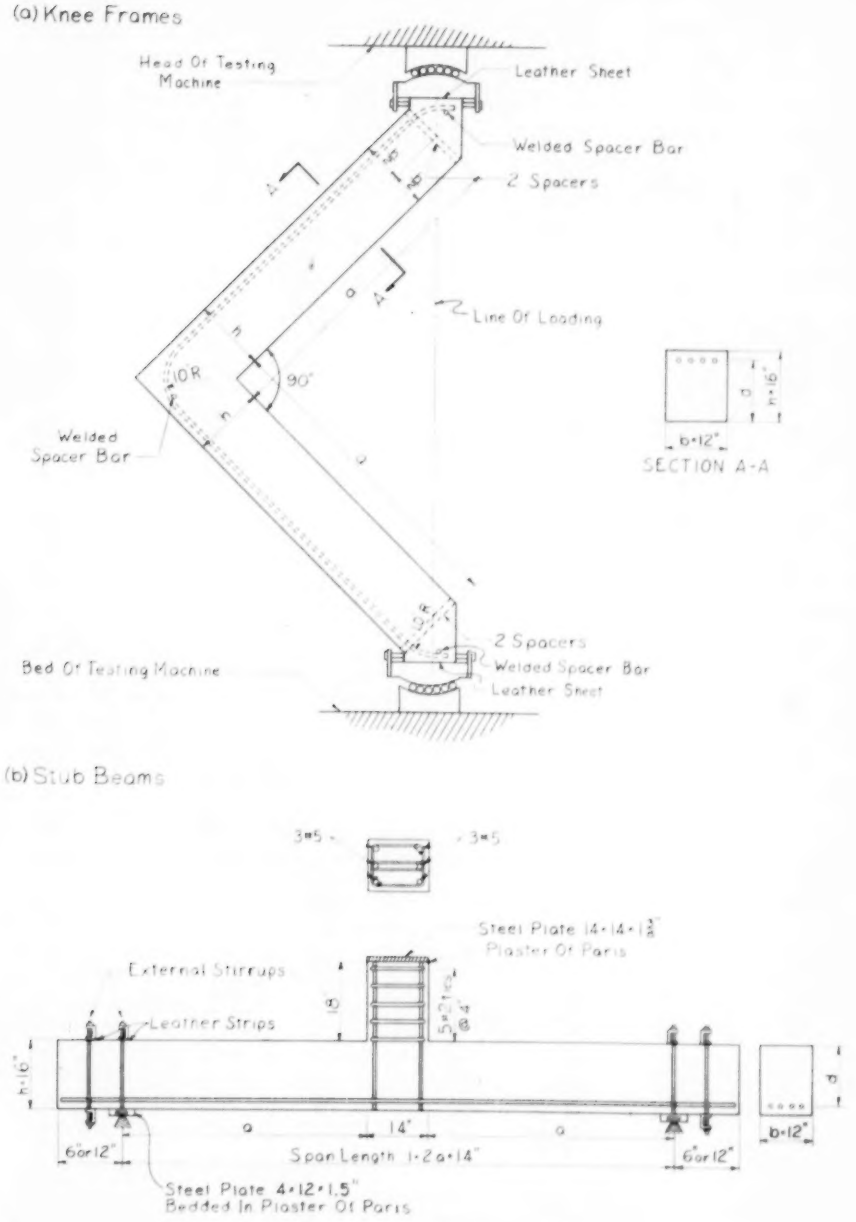

Fig. 1—Details of specimens

TABLE 2—DIMENSIONS OF SPECIMENS

Specimen*	Width b, in.	Depth h, in.	Effective depth d,† in.	Shear span a, in.	Reinforcing bars	Percentage of reinforcement
Stub beams						
B14B2	12.00	16.00	14.50	14.0	2 #6 4 #7	1.85
E2	12.00	16.13	14.75	14.0	2 #4 2 #5	0.57
A4	12.00	16.00	14.25	14.0	2 #7 4 #8	2.50
B4	12.00	16.00	14.50	14.0	2 #6 4 #7	1.85
E4	12.00	16.00	14.50	14.0	5 #6	1.24
A6	12.00	16.00	14.00	14.0	2 #8 5 #9	3.83
B6	12.00	16.00	14.50	14.0	2 #6 4 #7	1.85
B21B2	12.00	16.00	14.44	21.0	2 #6 4 #7	1.86
E2	12.00	16.00	14.75	21.0	2 #4 2 #5	0.57
A4	12.00	16.00	14.50	21.0	2 #7 4 #8	2.46
B4	12.00	16.00	14.50	21.0	2 #6 4 #7	1.85
E4	12.00	16.00	14.38	21.0	5 #6	1.24
E4R	12.00	16.00	14.50	21.0	5 #6	1.24
F4	12.00	16.00	14.56	21.0	2 #7 1 #8	1.17
G4	12.00	16.00	14.69	21.0	2 #4 2 #5	0.58
A6	12.00	16.00	14.00	21.0	2 #8 5 #9	3.83
B6	12.00	16.00	14.75	21.0	2 #6 4 #7	1.82
B28B2	12.00	16.00	14.25	28.0	2 #6 4 #7	1.88
E2	12.13	16.00	14.63	28.0	2 #4 2 #5	0.57
A4	12.00	16.00	14.50	28.0	2 #7 4 #8	2.46
B4	12.00	16.00	14.50	28.0	2 #6 4 #7	1.85
E4	12.00	16.00	14.50	28.0	5 #6	1.24
A6	12.13	16.00	13.88	28.0	2 #8 5 #9	3.83
B6	12.00	16.00	14.50	28.0	2 #6 4 #7	1.85
B40B4	12.00	16.00	14.50	40.0	2 #6 4 #7	1.85
B56B2	12.00	16.00	14.50	56.0	2 #6 4 #7	1.85
E2	12.00	16.00	14.50	56.0	2 #4 2 #5	0.58
A4	12.00	16.00	14.75	56.0	2 #7 4 #8	2.41
B4	12.00	16.00	14.50	56.0	2 #6 4 #7	1.85
E4	12.00	16.00	14.50	56.0	5 #6	1.24
A6	12.13	16.00	14.00	56.0	2 #8 5 #9	3.79
B6	12.00	16.00	14.63	56.0	2 #6 4 #7	1.83
B70B2	12.00	16.00	14.38	70.0	2 #6 4 #7	1.86
A4	12.00	16.00	14.50	70.0	2 #7 4 #8	2.46
A6	12.00	16.00	14.00	70.0	2 #8 5 #9	3.83
B84B4	12.00	16.00	14.31	84.0	2 #6 4 #7	1.88
B113B4	12.00	16.00	14.38	113.0	2 #6 4 #7	1.86
B4R	12.00	16.00	14.50	113.0	2 #6 4 #7	1.86
Knee frames						
F21B2	12.00	16.50	14.50	21.6	2 #6 4 #7	1.88
B2R	12.00	16.50	14.50	21.6	2 #6 4 #7	1.88
E2	12.00	16.50	14.75	21.6	2 #5	0.34
B4	12.00	16.50	14.50	21.6	2 #6 4 #7	1.88
B4R	12.00	16.50	14.50	21.6	2 #6 4 #7	1.88
C4	12.00	16.25	14.50	21.6	2 #5 5 #6	1.59
C4R	12.00	16.25	14.75	21.6	2 #5 5 #6	1.56
D4	12.00	16.25	14.50	21.6	7 #5	1.22
E4	12.25	16.50	15.00	21.6	2 #5 2 #6	0.80
F4	12.00	16.50	14.50	21.6	3 #6	0.75
G4	12.00	16.50	14.50	21.6	2 #5	0.35

*Designation of specimens may be explained by reference to Table 1: First letter stands for type of specimen —F = knee frame, B = stub beam; first number gives length of shear span in in.; second letter designates nominal percentage of reinforcement; second number designates nominal strength of concrete in ksi; and the letter R designates recast specimens. Thus F21B2R designates a knee frame with shear span of 21.6 in., nominal percentage of reinforcement of 1.89 percent, nominal concrete strength of 2000 psi, and it is the second specimen of this type.

†Distance from compression surface to centroid of tension reinforcement.

TABLE 2 (Continued)—DIMENSIONS OF SPECIMENS

Specimen*	Width b, in.	Depth h, in.	Effective depth d,† in.	Shear span a, in.	Reinforcing bars	Percentage of reinforcement
			Knee frames			
A6	12.00	16.50	14.50	21.6	3 #7 3 #8	2.36
B6	12.00	16.50	14.50	21.6	2 #6 4 #7	1.88
F38B2	12.00	16.50	14.25	38.6	2 #6 4 #7	1.91
E2	12.00	16.50	14.50	38.6	2 #6	0.50
B4	12.00	16.38	14.75	38.6	2 #6 4 #7	1.84
D4	12.13	16.38	15.00	38.6	2 #5 3 #7	1.32
E4	12.00	16.50	14.88	38.6	1 #6 2 #7	0.91
A6	12.00	16.50	14.00	38.6	3 #7 4 #8	2.91
B6	12.00	16.50	15.00	38.6	2 #6 4 #7	1.81
F55B2	12.00	16.50	14.50	55.6	2 #6 4 #7	1.88
E2	12.00	16.50	14.50	55.6	3 #5	0.52
A4	12.13	16.13	14.63	55.6	2 #7 3 #8	1.98
B4	12.00	16.50	15.00	55.6	2 #6 4 #7	1.81
D4	12.13	16.13	15.00	55.6	3 #5 4 #6	1.45
E4	12.13	16.88	15.25	55.6	4 #6	0.94
A6	12.00	16.50	13.75	55.6	4 #7 4 #8	3.32
B6	12.00	16.50	14.50	55.6	2 #6 4 #7	1.88
F70B2	12.00	16.50	14.25	70.0	2 #6 4 #7	1.91
A4	12.00	16.13	14.25	70.0	2 #5 4 #8	2.16
A6	12.00	16.50	14.50	70.0	6 #9	3.34
F84B4	12.00	16.38	14.75	84.0	2 #6 4 #7	1.84
F113B4	12.00	16.75	14.50	113.0	2 #6 4 #7	1.87

*, † See footnotes, opposite page.

lowered into the form and held in place by stirrup spacers at the end of each leg. The steel was secured in position by wiring to the forms.

The stub beams were cast in steel forms in the same position as tested. The longitudinal reinforcement was wired to steel chairs thus forming a reinforcing mat supported from the bottom of the forms. The steel for the column stub was tied into a cage and inserted into the forms.

Two to eight batches of concrete were required for each specimen. They were placed in uniform horizontal layers compacted with internal rod vibrators. On the day following casting the forms were removed and the specimens subjected to moist curing for an additional 6 days. Seven days after casting the specimens were stored in the laboratory and allowed to dry until testing at the age of approximately 28 days.

Three to five 6 x 12-in. control cylinders were made from each batch. Standard steel molds were filled in three layers compacted with an internal rod vibrator. A few hours after casting the cylinders were capped with neat cement paste. The cylinders were removed from the forms cured, stored and tested with the corresponding specimens.

TABLE 3—GRADING OF SAND AND GRAVEL*

Sieve size	Percent retained	
	Sand	Gravel
1½ in.	0.0	0.0
¾ in.	0.0	30.6
⅜ in.	0.0	85.2
No. 4	1.6	97.8
No. 8	11.8	99.9
No. 16	26.9	100.0
No. 30	56.8	100.0
No. 50	91.2	100.0
No. 100	98.4	100.0
Fineness modulus	2.87	7.14

*Average for eight lots of sand and gravel.

Test equipment and procedures

The knee frames were tested in a 3,000,000-lb hydraulic testing machine. A knee frame ready for testing is shown in Fig. 2 and a schematic diagram of the testing arrangement is

TABLE 4—STRENGTH OF CONCRETE AND STEEL

Specimen	Average strength of concrete* f'_c, psi	Average strength of reinforcement		Specimen	Average strength of concrete* f'_c, psi	Average strength of reinforcement	
		Yield point† f_y, ksi	Ultimate,† ksi			Yield point† f_y, ksi	Ultimate,† ksi
Knee frames				**Stub beams**			
F21B2	1470	54.6	99.8	B14B2	2120	67.6	126.2
B2R	2040	54.6	99.8	E2	1840	67.5	122.1
E2	1750	65.8	118.5	A4	3270	61.7	119.7
B4	4310	54.8	99.0	B4	3820	58.6	107.7
B4R	4230	54.8	99.0	E4	4190	61.2	113.3
C4	3860	62.9	114.4				
C4R	4480	64.8	113.8	A6	6590	65.0	126.7
D4	4570	65.6	116.4	B6	6780	65.9	125.3
E4	4450	62.6	114.2				
F4	4350	62.0	114.2	B21B2	2010	63.4	121.2
G4	4420	65.5	116.0	E2	1640	67.2	125.0
A6	7020	54.6	101.4	A4	4320	58.7	106.7
B6	6520	54.8	100.0	B4	3930	61.3	111.0
				E4	3510	62.4	113.9
F38B2	1800	54.2	98.4	E4R	4630	60.4	114.8
E2	2050	56.3	101.6	F4	4560	66.2	126.1
				G4	4580	67.8	117.2
B4	4550	55.9‡	100.9‡				
D4	3900	53.4‡	93.2‡	A6	6570	64.9	125.1
E4	4660	53.4‡	94.0‡	B6	6600	63.4	125.3
A6	6610	52.8	102.2	B28B2	2130	68.3	125.8
B6	6030	54.9	101.3	E2	1990	67.2	122.7
F55B2	1720	54.3	98.4	A4	3990	48.2‡	84.7‡
E2	2000	64.6	116.9	B4	4690	64.0	120.8
				E4	4800	62.2	112.7
A4	3830	58.8	107.0				
B4	4280	55.7	104.0	A6	6840	66.0	127.7
				B6	6360	65.5	127.2
F55D4	3710	62.7	113.7				
E4	4110	61.3	113.2	B40B4	5040	54.8	98.3
A6	6100	55.0	101.5	B56B2	2130	68.3	126.1
B6	6340	54.5	99.3	E2	2130	67.0	121.7
F70B2	2090	55.5‡	104.2‡	A4	3620	47.8‡	83.9‡
				B4	3950	63.9	123.3
A4	4210	54.5‡	98.0‡	E4	4120	62.2	112.7
A6	5610	51.4‡	98.6‡	A6	5780	63.6	129.0
				B6	6630	67.6	127.8
F84B4	4300	55.3	99.0				
				B70B2	2370	67.0	126.3
F113B4	3770	54.5	100.7				
				A4	3950	63.2	124.2
				A6	6520	63.1	126.5
				B84B4	3950	67.4	126.7
				B113B4	4730	68.0	126.8
				B4R	4160	50.0§	89.7

*Data obtained from tests of 6 x 12-in. concrete cylinders cast and cured together with the corresponding specimen. Three to five cylinders were made from each batch; one specimen required two to eight batches of concrete.

†One tension specimen was tested from each length of bar; the values given are weighted averages for all bars in the specimen. All bars complied with ASTM A 15-52T for hard-grade billet steel except when noted otherwise.

‡One to four bars of intermediate-grade steel.

§Bars with yield points 48.0-52.0 selected to insure flexural failure.

shown in Fig. 1a. The load was applied to the specimen through two special roller bearing blocks allowing practically free rotation in the plane of the frame. The center of rotation of each block was located in the middle of the bearing surface of the loading stub. The top bearing block was attached to the head of the testing machine and the bottom block rested on the bed of the machine. The head had a spherical block which was wedged after a small increment of load at the beginning of each test.

The beams were tested in a 300,000-lb screw type testing machine.* The arrangement

*Specimens B14A6 and B14B6, which did not fail at the load of 300,000 lb were retested in the 3,000,000-lb machine used in the tests of knee frames.

for testing is shown schematically in Fig. 1b. The specimen was supported on the bed of the testing machine through two knife-edge supports bearing on two 4 x 12 x 1.5-in. bearing plates. The load was applied with the head of the testing machine to a steel plate on the top of the column stub through a spherical block centered with respect to the supports and the beam width.

The load was applied in several increments to collapse of the specimen. After each increment, the load was stabilized and measurements were taken of deflections, steel strains, and concrete strains. In addition, the specimen was examined for crack formation and a pictorial record was kept of changes in the crack pattern. Testing of one specimen took from 2 to 6 hr.

Deflections were measured in the line of the load between the ends of the shear span in the knee frame tests and at midspan in the stub beam tests with deflectometers equipped with a 0.001-in. dial indicator.

TABLE 5—AVERAGE DIMENSIONS OF BARS*

Bar size	Area, sq in.	Deformations		
		Spacing, in.	Height, in.	Gap, in.
Knee frames				
5	0.302	0.410	0.041	0.124
6	0.433	0.467	0.043	0.131
7	0.599	0.534	0.055	0.159
8	0.772	0.664	0.055	0.155
9	0.968	0.627	0.070	0.170
Stub beams				
4	0.198	0.319	0.018†	0.078
5	0.309	0.397	0.034	0.107
6	0.430	0.464	0.039	0.128
7	0.590	0.517	0.047	0.143
8	0.773	0.658	0.052	0.153
9	0.977	0.706	0.059	0.174

*Dimensions, measured as specified in ASTM A 305-50T, are averages for at least ten bars; at least five bars of each size were taken at random from each lot. All bars had transverse deformation perpendicular to bar axis.

†Specified minimum is 0.020 in. All other measurements of deformations met the ASTM requirements.

Steel strains were measured with a 4-in. Berry mechanical strain gage. The gage lines were established by drilling small holes in the longitudinal steel through core holes formed in the sides of the specimen during casting. Steel strains were measured on both sides and both halves of each specimen. The gage lines were either continuous throughout the length of the shear span or were located only near the critical sections.

Concrete strains were measured on the compressive surfaces with SR-4 electric strain gages having gage lengths of 1 or 6 in. Strain gages were located at the center of the compression surface adjacent to the critical sections and at various points along the shear span.

TEST DATA

The results of the tests are summarized in Table 6 which includes the loads at the formation of the initial diagonal tension crack P_c, the load at first yielding of the longitudinal reinforcement P_y, the failure load P_u, and the deflections, steel strains, and concrete strains at failure load. In addition, the table includes the nominal shearing stress v_c, the ratios of the nominal shearing stresses v_c and v_u to the strength of concrete f_c', and modes of failure.

The diagonal tension cracking load was determined by visual observations; it is that load at which a definite diagonal tension crack was first observed. The cracking loads are more accurate for long than for short specimens because in long beams the diagonal tension cracks formed suddenly as opposed to gradual formation in short specimens. In beams B56B6 and B70A6 the diagonal tension crack was observed to form at a load slightly lower than the failure load, but final failure was characteristic of diagonal tension failures. Therefore, for these two specimens the values given in Table 6 as diagonal tension cracking loads are the actual failure loads.

The yield load was determined from the load-steel strain and load-deflection curves; it corresponds to first yielding of the longitudinal steel.

TABLE 6—TEST RESULTS

Specimen	Diagonal tension cracking			Yield load P_y, kips	Failure			Deformations at P_u		
	load P_{cr}, kips	stress v_{cr}, psi	$\frac{v_{cr}}{f_c'}$		load P_u, kips	$\frac{v_u}{f_c'}$	Mode*	Max. defl., in.	Max. strains, millionths	
									steel	concrete
Knee frames										
F21B2	42.0	195	0.133	——	65.5	0.207	S	0.199	700†	7770
B2R	55.0	255	0.125	——	90.0	0.205	S	0.290	950	5700
E2	——	——	——	50.0	55.5	0.145	F	0.460	6900‡	5980
B4	79.0	367	0.085	——	117.5	0.127	S	0.199	1110†	4950†
B4R	75.0	348	0.082	——	130.0	0.143	S	0.154	1200	4950
C4	70.0	325	0.084	——	89.0	0.107	S	0.075	1110†	1800
C4R	80.0	365	0.082	——	141.0	0.144	S	0.268	1700†	7400
D4	70.0	325	0.071	——	141.5	0.144	S	0.250	1900†	3260†
E4	70.0	308	0.069	110.0	129.0	0.127	S	0.358	2190‡	5600†
F4	69.5	323	0.074	105.0	109.0	0.116	S	0.460	8100	4600
G4	——	——	——	54.0	69.5	0.073	F	0.695	19900‡	6400†
A6	90.0	418	0.060	——	200.0	0.132	S	0.204	1520	5700
B6	90.0	418	0.064	——	198.0	0.141	S	0.226	1640†	5300†
F38B2	36.0	167	0.093	——	36.0	0.093	D	0.390	720†	3380†
E2	23.5	109	0.053	27.5	29.2	0.066	S	0.500	2240‡	5900
B4	55.0	251	0.055	——	55.0	0.055	D	0.252	900	3860
D4	53.8	239	0.061	——	53.8	0.061	D	0.282	1250†	6070
E4	47.0	213	0.046	——	47.0	0.046	D	0.309	1450†	3190
A6	70.0	337	0.051	——	75.0	0.055	S	0.290	1000†	3550†
B6	55.0	247	0.041	——	63.0	0.047	S	0.300	1040	4200
F55B2	30.0	139	0.081	——	30.0	0.081	D	0.322	980	3350†
E2	——	——	——	20.0	25.0	0.058	F	1.130	4100‡	6000†
A4	48.0	218	0.057	——	48.0	0.057	D	0.441	1340†	3170
B4	40.0	180	0.042	——	40.0	0.042	D	0.335	1100†	2300†
D4	40.0	178	0.048	——	40.0	0.048	D	0.406	1300†	1160†
E4	35.0	153	0.037	40.0	41.4	0.044	S	0.637	2340‡	3190
A6	60.0	294	0.048	——	60.0	0.048	D	0.436	930	2110
B6	45.0	209	0.033	——	45.0	0.033	D	0.388	950†	1560†
F70B2	29.0	134	0.064	——	29.0	0.064	D	0.585	1420†	4150†
A4	45.0	208	0.049	——	45.0	0.049	D	0.700	1250†	4150†
A6	55.0	255	0.046	——	55.0	0.046	D	0.820	1160†	2600†
F84B4	41.5	189	0.044	37.5	41.5	0.044	D	1.479	4430‡	6600
F113B4	——	——	——	29.5	30.5	0.038	F	3.417	9230‡	5510
Stub beams										
B14B2	70.0	230	0.108	——	165.0	0.256	S	0.121	1150†	1360
E2	60.0	194	0.105	——	125.0	0.219	S	0.157	2170†	1300†
A4	100.0	334	0.102	——	230.0	0.235	S	0.144	1050†	——
B4	100.0	328	0.086	——	225.0	0.194	S	0.115†	1300†	1520†
E4	105.0	345	0.083	——	230.0	0.180	S	0.150†	1910†	1700†
A6	125.0	425	0.065	——	405.0	0.209	S	0.118	1300†	1080
B6	100.0	328	0.048	——	350.0	0.170	S	0.155	2030†	1410†
B21B2	70.0	231	0.115	——	107.0	0.176	S	0.143	980†	4600†
E2	45.0	145	0.088	95.0	95.0	0.187	S	0.192	2480†	3400†
A4	90.0	296	0.069	——	235.0	0.179	S	0.220†	1460†	4720†
B4	80.0	263	0.067	——	178.0	0.149	S	0.175	1500†	2370†
E4	80.0	295	0.084	180.0	190.0	0.179	S	0.251	2380†	2960
E4R	80.0	263	0.057	180.0	195.0	0.138	S	0.231	2900	2660
F4	90.0	294	0.065	190.0	210.0	0.151	S	0.261	3860‡	4300†
G4	75.0	243	0.053	100.0	159.0	0.112	F	1.380	18200‡	4930‡
A6	99.0	337	0.051	——	260.0	0.135	S	0.183	1290†	1560†
B6	100.0	323	0.049	——	260.0	0.127	S	0.225	2100†	4050†

*S—Shear failure; F—flexural tension failure; D—diagonal tension failure.
†Extrapolated values.
‡The value shown is the last reading taken (at a load lower than P_{ult}).

TABLE 6 (Continued)—TEST RESULTS

Specimen	Diagonal tension cracking			Yield load P_y, kips	Failure			Deformations at P_u		
	load P_c, kips	stress v_c, psi	$\frac{v_c}{f_c'}$		load P_u, kips	$\frac{v_u}{f_c'}$	Mode*	Max. defl., in.	Max. strains, millionths	
									steel	concrete
Stub beams										
B28B2	55.0	184	0.086	——	90.0	0.141	S	0.239	1080†	6540
E2	40.0	129	0.065	——	58.0	0.094	S	0.239	1950†	4870†
A4	68.0	223	0.056	——	145.0	0.119	S	0.243	1190†	5140†
B4	60.0	197	0.042	——	115.0	0.081	S	0.190	1220†	2650†
E4	55.0	181	0.038	——	120.0	0.082	S	0.253	2050†	3780†
A6	95.0	322	0.047	——	150.0	0.074	S	0.193	850†	1990
B6	80.0	263	0.041	——	145.0	0.075	S	0.230	1490†	2940
B40B4	69.5	228	0.045	——	70.0	0.046	S	0.199	1110†	1960‡
B56B2	45.0	148	0.070	——	45.0	0.070	D	0.282	980†	2140†
E2	30.0	99	0.047	34.3	35.8	0.055	S	0.665	5690‡	4510
A4	62.0	200	0.055	——	62.0	0.055	D	0.270†	1060†	1670†
B4	55.0	181	0.046	——	55.0	0.046	D	0.274	1210†	2030
E4	49.0	161	0.039	——	49.0	0.039	D	0.322	1750†	1330†
B56A6	80.0	269	0.047	——	80.0	0.047	D	0.280	1160†	1720†
B6	61.5	202	0.030	——	61.5	0.030	D	0.316	1310	1650†
B70B2	40.0	132	0.056	——	40.0	0.056	D	0.521	1330	3210
A4	59.5	195	0.049	——	59.5	0.049	D	0.480†	1330†	3460
A6	80.0	272	0.042	——	80.0	0.042	D	0.509	1250†	1880†
B84B4	50.0	166	0.042	——	50.0	0.042	D	0.776	1810†	3110
B113B4	46.9	155	0.033	——	46.9	0.033	D	1.540	2240†	3600
B4R	——	——	——	35.0	38.0	0.030	F	2.830	6380‡	3950‡

*, †, ‡ See footnotes, opposite page.

The failure load is the maximum load reached during the test. Several specimens continued to carry some load after the maximum had been reached; the test was usually continued until the load-carrying capacity of the specimen was only a small fraction of the maximum load. However, had the maximum load been maintained on the specimen for a short period of time, complete failure would have occurred under the maximum load; therefore, the maximum load is referred to as failure load. In beam B56E4 a diagonal tension crack formed and failure followed after a small increase of load, but the load could not be stabilized at load levels higher than the diagonal tension cracking load. Therefore, for this specimen the value given in Table 6 as the failure load is the actual diagonal tension cracking load.

The deflections for knee frames are the decrease of the distance between the center points of the end sections of the upper and lower legs. The deflections for the stub beams are the values measured at midspan with respect to the supports. In most tests, the deflections were recorded at the instant when the maximum load was reached.

The strains are those which occurred at or near the critical sections. Most of the strain data in Table 6 are extrapolated from readings at the last increment before the failure load.

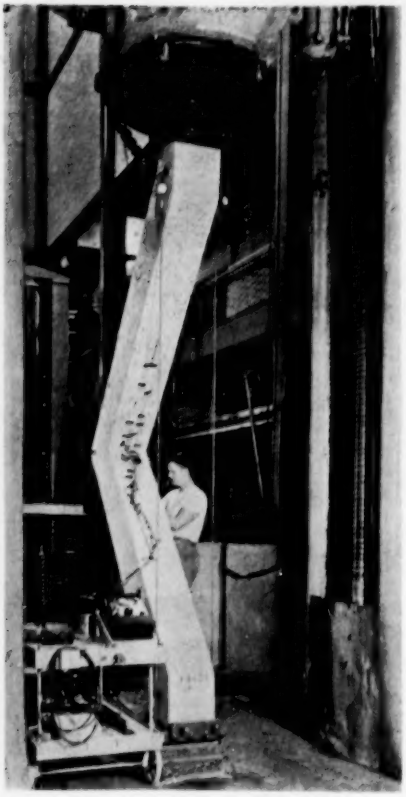

Fig. 2—Knee frame F84B4

The nominal shearing stresses at diagonal tension cracking, v_c, and at failure, v_u, were obtained from the respective loads as $8V/7bd$ in which V is the shear corresponding to the appropriate value of load P; b is the width, and d is the effective depth of the cross section.

All specimens failed by one of the three following modes of failure: diagonal tension failure designated in Table 6 as D, shear compression failure designated as S, or flexural failure designated by F. Diagonal tension failure is defined as a failure by formation of the first diagonal tension crack at the maximum load. The shear failure is defined as a failure by destruction of the compressive zone of concrete at the critical section above a diagonal tension crack; it may occur either before or after yielding of the longitudinal reinforcement but always at relatively small deflections. Flexural failure is defined as a failure by crushing of the concrete in compression either before formation of diagonal tension cracks or after very large deflections; the first type may occur either before or after yielding of longitudinal reinforcement, but the second type can occur only after considerable yielding.

The stub beams were supported on two knife edge supports and the knee frames were loaded through special roller-bearing shoes. If the knife edges prevented horizontal movements, the beams would have acted as two-hinged arches and their capacity would have been higher than that of simply supported beams. Similarly, if the bearing shoes did not permit free rotation, the capacity of the knee frames would have been increased. A study of the test data indicates that, if present, the effects of improper functioning of supports were negligible in all specimens which failed before yielding of the steel and in all long specimens. In short specimens which failed after the yielding of reinforcement, the improper functioning of supports may have increased the ultimate load by 5 to 15 percent, and the ultimate strength of B21G4 may have increased even more.

Behavior under load

The behavior of the knee frames and the stub beams is illustrated in Fig.

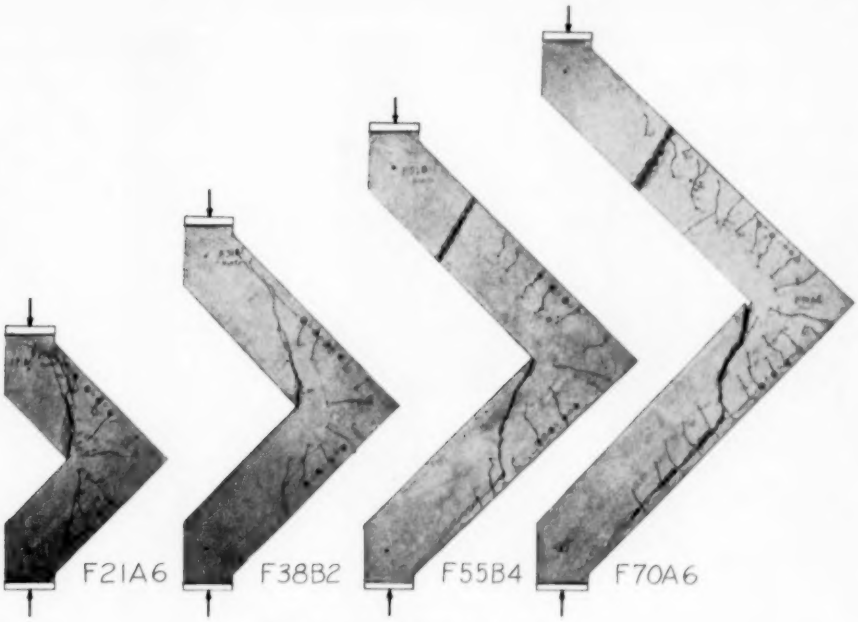

Fig. 3—Knee frames after failure

3-7. Fig. 3 and 4 show typical knee frames and stub beams of various lengths of shear span after failure. Typical load-steel strain curves are shown in Fig. 5, typical strain distributions along the shear span are illustrated in Fig. 6, and typical load-deflection curves are shown in Fig. 7.

Before the formation of the first diagonal tension crack, the behavior of all specimens was characteristic of reinforced concrete flexural members. The steel strains were smallest near the supports and increased toward the critical section (Fig. 5); the steel strain distribution along the shear span was approximately linear (Fig. 6, $P = 45$ kips).

In all specimens which failed in diagonal tension or in shear, one or more distinct diagonal tension cracks formed at some stage of loading. The appearance of such cracks may be seen in Fig. 3 and 4. The manner of formation of the diagonal tension cracks varied with varying length of shear span. In short shear spans the diagonal cracks formed progressively, usually starting at the level of the steel, approximately one beam depth from the critical section. In very short beams, such as B14B6, usually more than one diagonal crack formed in each shear span before failure. In longer specimens, *e.g.* B28B6, one diagonal crack formed in each shear span. In specimens with very long shear spans, *e.g.* F38B2, and B56B4, only one diagonal crack formed; the crack formed suddenly, starting at or near an existing tension crack and extending both toward the steel and the critical section. It intersected the steel approximately in the middle of the shear span.

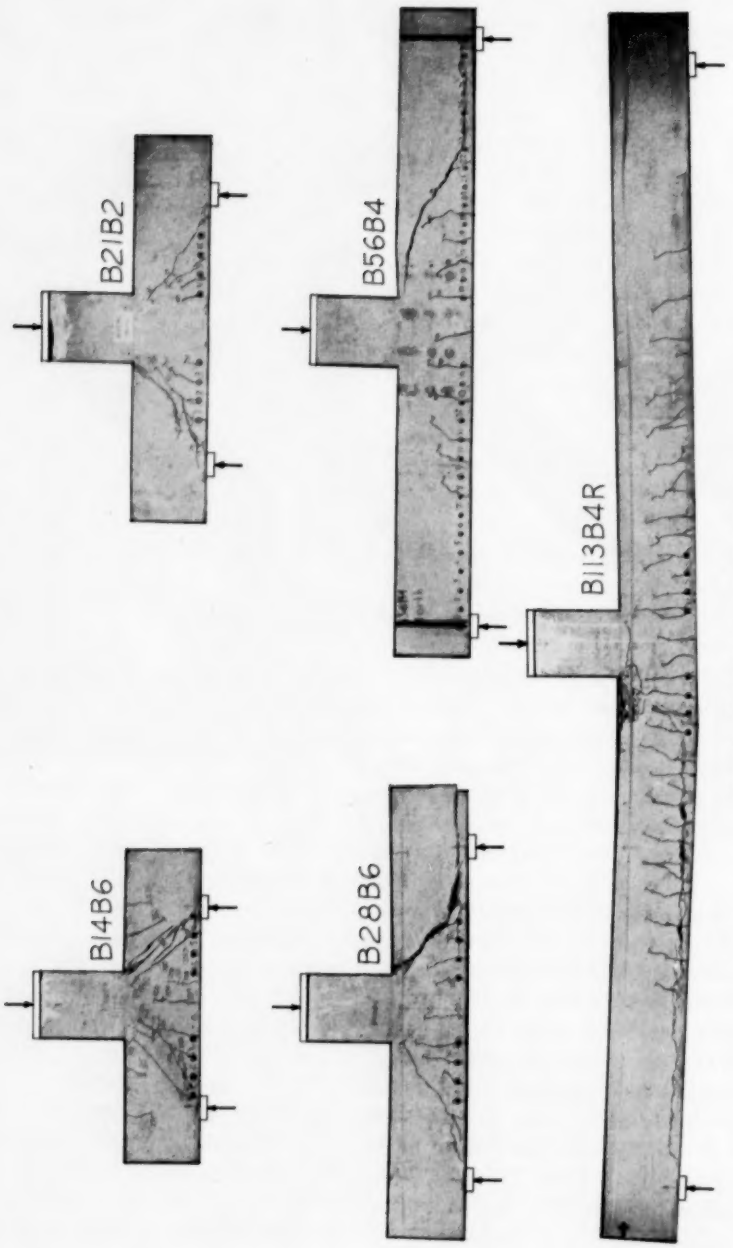

Fig. 4—Stub beams after failure

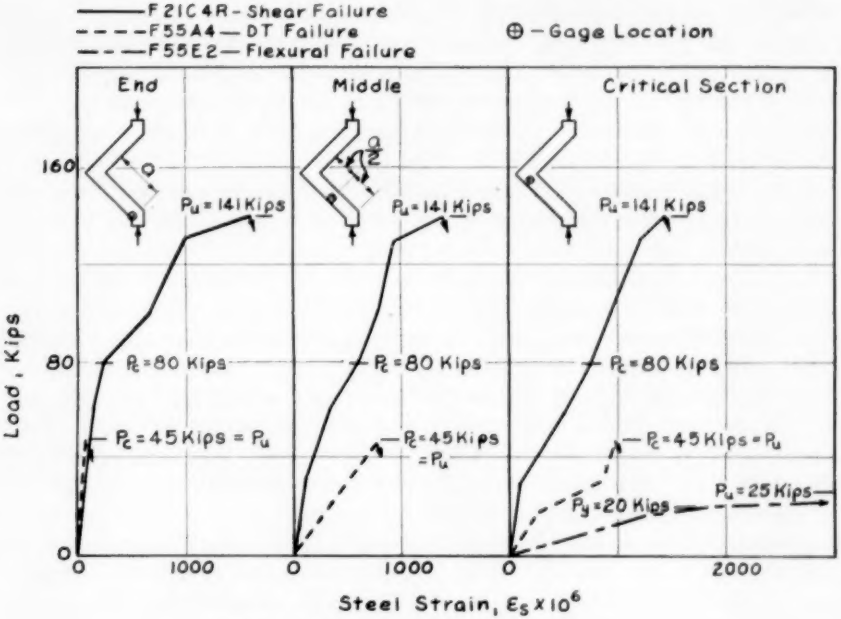

Fig. 5—Typical load-steel strain curves

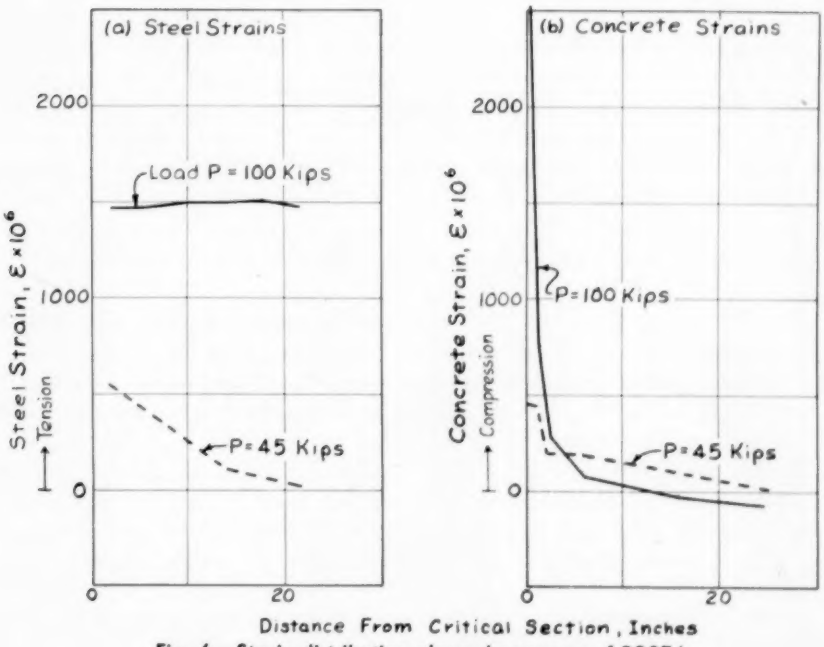

Fig. 6—Strain distribution along shear span of B28E4

Diagonal tension failures—In long specimens the sudden formation of the first diagonal tension crack was usually accompanied by complete collapse; the failure of such beams is defined as a diagonal tension failure. Frames F38B2, F55B4, F70A6, and beam B56B4 in Fig. 3 and 4 are examples of diagonal tension failures. The sudden, brittle nature of this type of failure is illustrated in Fig. 5 and 7.

Frame F84B4 failed in diagonal tension after the longitudinal steel had yielded. In all other specimens, the diagonal tension crack formed while the steel stresses were elastic.

Shear compression failures—In beams in which the formation of the diagonal tension crack did not cause failure, redistribution of internal stresses was observed. The stresses and strains in the longitudinal reinforcement increased more rapidly at sections away from than at sections near the critical so that after some increase in load, the distribution of steel strains was approximately uniform between the critical section and the intersection of the diagonal tension crack with the longitudinal steel (F21C4R in Fig. 5 and $P = 100$ kips in Fig. 6). The maximum compressive stresses and strains in the concrete increased considerably at the critical section; at sections near the support tensile stresses were observed on the "compression" surface of the specimens ($P = 100$ kips in Fig. 6). The concentration of concrete compressive stresses at the critical section was caused by concentrated rotations at

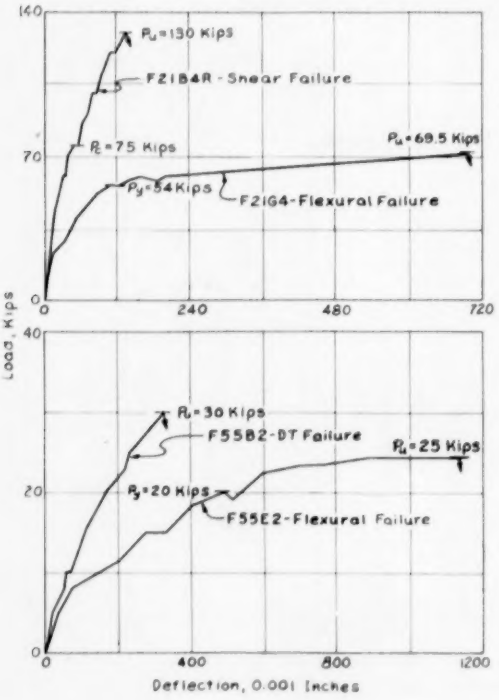

Fig. 7—Typical load-deflection curves

the compression end of the diagonal crack; the tensile stresses on the "compression" surface were caused by arch action present after the formation of the diagonal crack.

The concentration of compressive strains at the critical section led to a premature crushing of the compression zone of concrete and thus to failure before the flexural capacity was reached. This type of failure is defined as shear compression failure; it occurred in short specimens (F21A6, B21B2, and B28B6 in Fig. 3 and 4). The shear compression failures were sudden; although stiffness of the specimen decreased slightly with the formation of the diagonal crack, the deflection at failure was small (Fig. 7).

The beams with 14-in. shear spans appeared to fail by formation of a new diagonal crack. This crack was observed first in the compression zone near the critical section, and at failure extended toward the top surface and downward toward the support. In this connection, it is interesting to note that the maximum concrete strain observed in these very short beams was 0.0017, considerably less than the maximum concrete strain in specimens in which crushing of the compression zone was observed. It appears then that very short specimens are characterized by their own mode of failure designated by Laupa[3] as failure by "shear proper." However, in this report no distinction is made between the "shear proper" and shear failures; both types are designated as shear compression failures.

Shear compression failures were observed both before or after yielding of the longitudinal reinforcement. They were sudden and always accompanied by a substantial decrease of the load-carrying capacity.

Flexural failures—In five specimens, diagonal tension cracks did not form. All of them failed in flexure by excessive yielding of the reinforcement followed by crushing of the concrete either in the compression zone at the critical section (B113B4R in Fig. 4) or in the knee along the bar bend. Beam B21G4 failed after the formation of diagonal cracks; however, its failure was classified as flexural because exceedingly large deformations were observed before the crushing of concrete occurred.

All flexural failures were gradual and were accompanied by yielding of the reinforcement.

Relationship between modes of failure

An examination of Table 6 reveals that short specimens failed in shear compression after the formation of a diagonal tension crack, medium long specimens failed in diagonal tension, and the longest specimens failed in flexure. Flexural failures were observed also in shorter specimens F21E2, F21G4, F55E2, and B21G4, all of which had a percentage of longitudinal reinforcement smaller than 0.6 percent, thus illustrating that reinforced concrete flexural members with very small amounts of longitudinal reinforcement fail in flexure even though no shear reinforcement is provided. This effect of the percentage of reinforcement on the mode of failure of beams without web reinforcement was discussed in a previous paper.[1]

The effect of the length of shear span on the mode of failure is illustrated

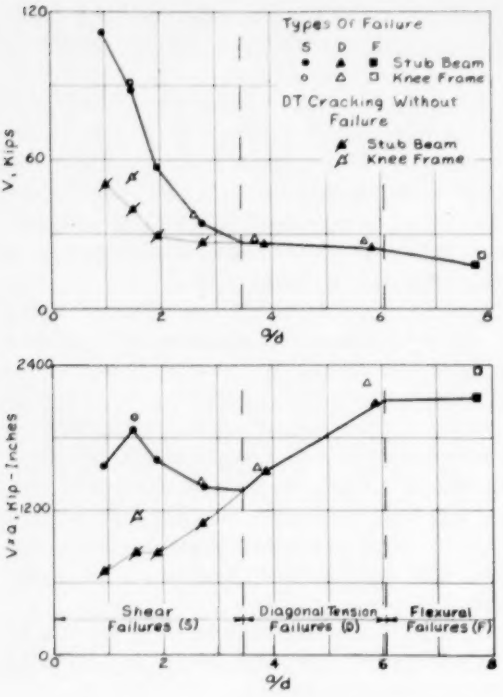

Fig. 8—Variations of modes of failure with a/d (f_c' = 4000 psi, p = 1.89 percent)

in Fig. 8 which includes test data for specimens with nominal concrete strength of 4000 psi and nominal percentage of reinforcement of 1.89 percent. In the upper part of the figure, the shear at failure and at first diagonal tension cracking, V, is plotted as a function of the ratio of the shear span to the effective beam depth (a/d ratio). The lower part of the figure includes the same data, but the shear V is multiplied by the length of the shear span, a.

The data for the stub beams are represented by full symbols and are connected by two lines: a heavy line representing failure conditions, and a light line representing diagonal tension cracking before failure. It can be seen that in beams with a/d ratios smaller than about 3.4, a diagonal tension crack formed first and shear compression failure followed at a higher load. The difference between the diagonal tension cracking load and the shear compression failure load decreased with increasing a/d ratio. Beams with a/d ratios larger than 3.4 but smaller than about 6.1 failed in diagonal tension and those with a/d ratios larger than 6.1 failed in flexure without the presence of a diagonal tension crack.

The numerical values of the limiting a/d ratios quoted in the previous paragraph vary with the properties of the specimen. The data in Fig. 8 illustrate only qualitatively that shear affects the strength of beams with small and moderately large a/d ratios but is not a factor in long specimens. It is

also interesting that the failure load of beams failing in diagonal tension (moderately large a/d ratios) is almost independent of the a/d ratio.

Earlier investigations[1,2,3] have suggested that the moment Va at the critical section of simple beams failing in shear is independent of the length of the shear span. On the contrary, data from this investigation indicated a consistent increase of the shear moment capacity with decreasing length of the shear span, this increase being especially large as the a/d ratio decreased from 2 to 1.5. Whether this increase is characteristic of beams with column stubs, or whether its lack in the earlier investigations resulted from some other causes, cannot be resolved on the basis of the available experimental evidence.

The data for the knee frames, represented in Fig. 8 by open symbols, illustrate that the qualitative effect of the a/d ratio on the mode of failure of the knee frames was the same as for stub beams.

Effects of axial compression

The qualitative behavior of the knee frames and the stub beams was the same; the appearance of specimens failing in the same manner was similar (Fig. 3 and 4), and the effects of variables on the mode of failure were qualitatively the same (Table 6). The quantitative effects of the presence of axial compression are illustrated in Fig. 8.

For short specimens, the axial compression increased considerably the shear at diagonal tension cracking but only slightly the shear at shear compression failure.* Consequently, the limit between the shear and diagonal tension failures was shifted to smaller a/d ratios as a result of the presence of axial compression. For longer specimens, however, the increase of shear at diagonal tension cracking was small and of the same order of magnitude as the increase of flexural capacity. Accordingly, the limit between the diagonal tension and flexural failures was unaffected by the presence of axial compression.

The diagonal tension cracking governs the capacity only for medium long specimens, that is in the range in which axial compression has only small effect. Since the shear compression strength, governing the capacity of short specimens, and the flexural strength, governing the capacity of very long specimens, appear to be affected only slightly by the presence of axial compression, it may be concluded that within the limits of this study the axial compression had only a negligible effect on the strength of reinforced concrete flexural members without web reinforcement.

The data included in this report are limited to stub beams with the axial force equal to zero and to knee frames with the axial compressive force equal to the shear force; thus the conclusions based on Fig. 8 must necessarily be limited to the axial compressive force to shear ratio of 1. What effect the axial compression has on the strength of members with different ratios of axial force to shear, cannot be determined on the basis of this series of tests alone.

*In the comparable specimens B21B4 and F21B4R shown in Fig. 8 the axial compression increased V_c by 32.5 percent, but V_u only by 3.2 percent.

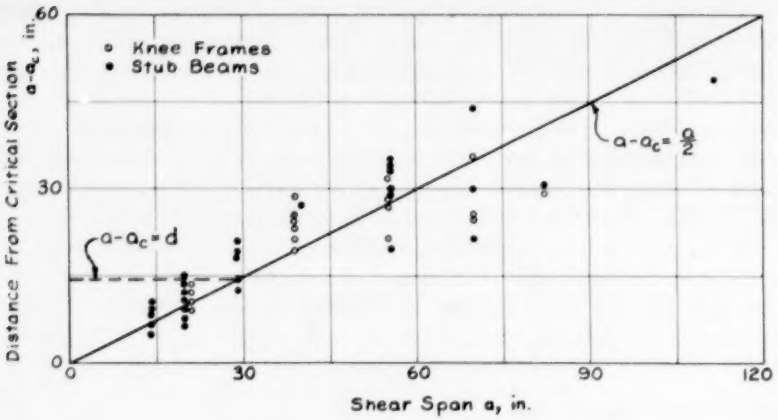

Fig. 9—Location of initial diagonal tension crack

ULTIMATE LOADS

Diagonal tension cracking

For the purposes of this paper a diagonal tension crack is defined as an inclined crack extending from the longitudinal reinforcement to the critical section and making an angle of approximately 45 deg with the longitudinal reinforcement at the level of the steel. The diagonal tension cracking load is the load at which the diagonal tension crack of the type defined above was first clearly observed.

The distance between the critical section and the intersection of the diagonal crack with the longitudinal steel is plotted in Fig. 9 against the length of the shear span. It may be seen that the point of intersection was usually located near the middle of the shear span, although for specimens with shear spans shorter than 30 in. a distance equal to the effective beam depth gives an almost equally good approximation. Since the diagonal crack started always near the point of intersection with longitudinal reinforcement, it will be assumed in the following that a diagonal tension crack forms at the distance a_c from the section of zero moment* governed by the following three relationships:

$$a_c = 0 \quad \text{for} \quad d \geqq a$$

$$a_c = a - d \quad \text{for} \quad a \geqq d \geqq \frac{a}{2}$$

$$a_c = \frac{a}{2} \quad \text{for} \quad \frac{a}{2} \geqq d$$

where a is the length of the shear span and d is the effective beam depth.

A diagonal tension crack forms because of the presence of excessively high principal tensile stresses. The location and inclination of the initial diagonal

*For stub beams the section of zero moment is at the support. For knee frames it is the section at which the load line intersects the centroidal axis (neglecting the presence of reinforcement).

TABLE 7—NOMINAL SHEARING STRESS AT DIAGONAL TENSION CRACKING*

Knee frames				Stub beams			
Specimen	Nominal shearing stress		Ratio $\frac{v_c \text{ test}}{v_c \text{ calc.}}$	Specimen	Nominal shearing stress		Ratio $\frac{v_c \text{ test}}{v_c \text{ calc.}}$
	v_c test, psi	v_c calc., psi			v_c test, psi	v_c calc., psi	
F21B2	198	197	1.01	B14B2	232	241	0.96
B2R	258	232	1.11	E2	196	224	0.87
B4	370	335	1.10	A4	336	301	1.12
B4R	351	332	1.06	B4	330	321	1.03
C4R	368	342	1.08	E4	347	339	1.02
D4	328	332	0.99				
E4	311	349	0.89	A6	427	424	1.01
F4	326	326	1.00	B6	330	429	0.77
A6	421	428	0.98	B21B2	234	202	1.16
B6	421	410	1.03	E2	148	150	0.99
F38B2	171	172	1.00	A4	299	295	1.01
E2	113	127	0.89	B4	266	270	0.99
				E4	298	239	1.25
B4	255	246	1.03	E4R	266	268	1.01
D4	243	218	1.12	F4	297	265	1.12
E4	217	208	1.04	G4	246	223	1.10
A6	341	313	1.09	A6	340	366	0.93
B6	251	276	0.91	B6	326	337	0.97
F55B2	145	150	0.96	B28B2	187	183	1.02
				E2	132	130	1.02
A4	224	209	1.07				
B4	186	213	0.87	A4	226	251	0.90
D4	184	190	0.97	B4	200	250	0.80
E4	159	174	0.91	E4	184	226	0.82
A6	300	279	1.08	A6	325	333	0.98
B6	215	243	0.89	B6	266	276	0.96
F70B2	141	151	0.94	B40B4	232	230	1.01
A4	215	202	1.06	B56B2	153	153	1.00
				E2	104	107	0.98
B6	262	255	1.03				
				A4	205	201	1.02
F84B4†	197	183	1.08	B4	186	187	0.99
				E4	166	171	0.97
	Average		1.007				
	Standard deviation		0.072	A6	274	264	1.04
				B6	207	225	0.92
				B70A2	137	148	0.93
				A4	200	195	1.03
				A6	277	259	1.07
				B84B4	172	169	1.02
				B113B4	163	163	1.00
					Average		0.994
					Standard deviation		0.093

All specimens:
Average 1.000
Standard deviation 0.083

*Test values include dead load.
†Yielding of steel observed before diagonal tension crack formation.

tension crack indicates that these principal tensile stresses are composed of three components: the flexural tension stress, the shearing stress caused by external shear, and the shearing stress caused by bond between the steel and concrete in the vicinity of tension cracks. It is believed that a diagonal tension crack begins to form when the resulting principal tensile stress reaches the value of the modulus of rupture of the concrete.

Starting from these considerations, Morrow[4] has suggested that the dimensionless ratio v_c/f_r' is a function of the dimensionless quantity

$$\frac{(M/V)_c}{npd}$$

In these expressions v_c is the nominal shearing stress at diagonal tension cracking, f_r' the modulus of rupture, n the modular ratio, p the ratio of longitudinal reinforcement, d the effective beam depth and $(M/V)_c$ the ratio of the moment around the centroid of compression force to the shear at the section where the crack starts. Thus for beams

$$(M/V)_c = a_c$$

and for knee frames

$$(M/V)_c \cong a_c - \frac{N}{V}\left(\frac{h}{2} - \frac{1}{8}d\right)$$

The correlation is plotted in Fig. 10 with the nominal shears v_c from Table 6 corrected for dead load and with the modulus of rupture f_c' taken as $9.5\sqrt{f_c'}$—an empirical relationship derived from Kesler's data.[5] The modular ratio n was computed as E_s/E_c with $E_c = 460\,f_c' + 1{,}800{,}000$. It can be seen in Fig. 10, that the test data form a well defined band which may be approximated by the following average curve:

$$\frac{v_c}{f_r'} = 0.15 + \frac{4}{\dfrac{(M/V)_c}{npd} + 10}$$

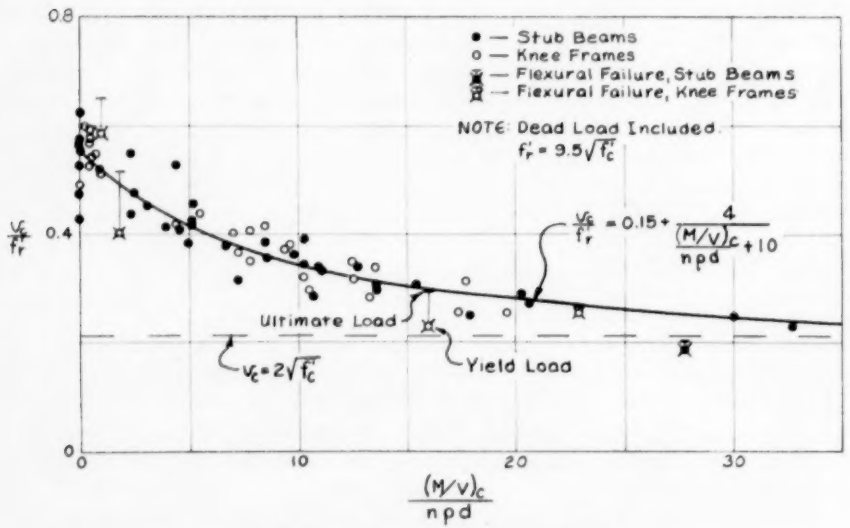

Fig. 10—Diagonal tension cracking

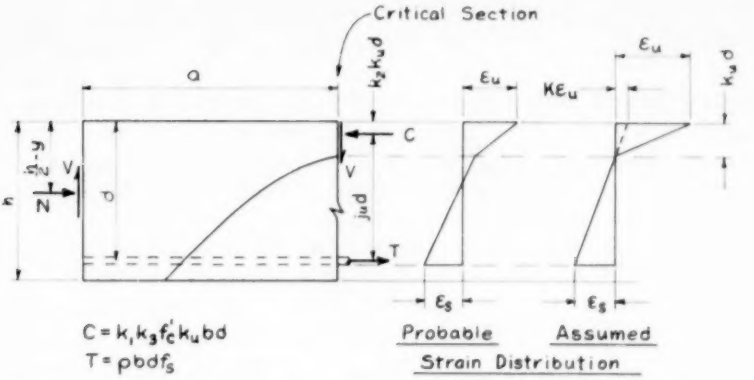

Fig. 11—Free body diagram and distribution of strains for shear failures

The nominal shearing stress at first diagonal tension cracking may, therefore, be expressed as

$$v_c = \left[1.425 + \frac{38}{\frac{(M/V)_c}{npd} + 10}\right]\sqrt{f_c'} \qquad (1)$$

where the concrete strength f_c' and the nominal shearing stress are expressed in psi.

Correlation between test data and Eq. (1) for individual specimens is shown in Table 7. The average ratio of test to calculated values is 1.007 for the knee frames and 0.994 for the stub beams. The average for all specimens is 1.000 and the corresponding standard derivation is 0.083. Eq. (1) takes into account effects of axial load on the diagonal tension cracking load, since it seems to be equally good for the knee frames and the stub beams.

Shear strength

Earlier studies[1-4] have shown that the load at shear failure may be correlated best on the basis of the failure moment at the critical section. With the notation of Fig. 11, the following analytical expression for the shear moment capacity may be derived from two equations of statical equilibrium.*

$$\frac{M_{ct}}{f_c'\,bd^2} = \frac{\alpha p f_s}{f_c'}\left(1 - \frac{k_2}{k_1k_3}\frac{\alpha p f_s}{f_c'}\right) \qquad (2)$$

where

$M_{ct} = Va + N\,(d - h/2 + y)$—moment around the centroid of tension reinforcement at the critical section.

*Equilibrium of longitudinal forces: $C = N + T$

Equilibrium of moments: $Va + N\,(d - h/2 + y) = C\left(1 - \frac{k_2}{k_1k_3}\frac{\alpha p f_s}{f_c'}\right)d$

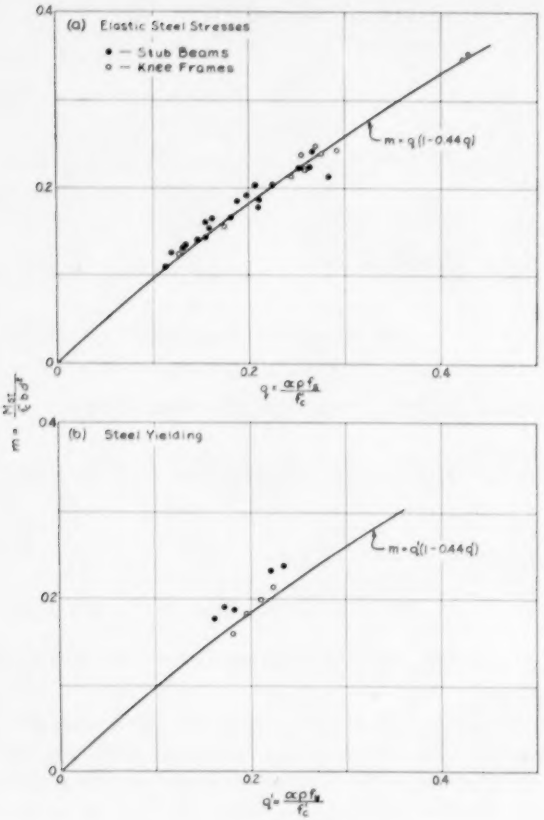

Fig. 12 — Shear compression failure

$$\alpha = 1 + \frac{N}{pbdf_s}$$

f_s = steel stresses at shear failure

The unknowns in Eq. (2) are the ratio of the properties of the stress block, k_2/k_1k_3, and the steel stress, f_s. The value of the ratio k_2/k_1k_3 may be calculated from Eq. (2) if the measured values of M_{st}, N, and f_s are substituted. In Fig. 12 measured values of $M_{st}/f_c'bd^2$ are plotted as functions of measured values of $\alpha pf_s/f_c'$. It can be seen that the data for specimens failing before yielding of reinforcement (Fig. 12a) form a narrow band closely approximated by a curve corresponding to $k_2/k_1k_3 = 0.44$.

For beams which failed after the reinforcement had yielded, the curve of $k_2/k_1k_3 = 0.44$ falls below the test data (Fig. 12b). This difference is believed to have resulted from several factors: a gentle slope of the stress-strain curves for the steel at stresses exceeding the yield point value, differences in the position of the neutral axis with respect to the top end of the diagonal tension

crack, and imperfect support conditions. The last factor, discussed earlier in this report, would explain also small differences between the knee frames and the stub beams.

The magnitude of steel stress at failure depends on the strain distribution at the critical section. The strain measurements show that the steel strains at shear compression failure are smaller than the strains which would be expected had a flexural failure taken place; on the other hand, the concrete strains at shear compression failure are approximately of the same order as expected for flexural failures. For a flexural failure, the strain distribution is linear; accordingly the strain observations and the conditions of statical equilibrium at shear failure can be fulfilled only if the strain distribution for a shear compression failure is nonlinear or discontinuous.

The diagonal tension crack ends at the critical section; this suggests the presence of concentrated rotations at the end of the crack. Accordingly, the strain distribution at shear compression failure probably takes the shape shown in Fig. 11 as the *probable strain distribution*. The probable strain distribution indicates that the neutral axis of beams failing in shear compression is located below the top of the diagonal crack so that a portion of the compressive stresses is transferred below the diagonal crack. However, in the specimens of this investigation no consistent differences were found in steel stresses measured at the critical section and at the intersection of the crack with the steel, so that the portion of compressive stresses transferred below the diagonal crack must have been small. It is believed, therefore, that the *assumed strain distribution* in Fig. 11 is sufficiently accurate for estimating the steel stresses at shear compression failure.

The assumed strain distribution and one condition of equilibrium at the critical section* give the following equation for the steel stress at shear compression failure:

$$f_s = \frac{1}{2} E_s K\epsilon_u \left[-1 + \sqrt{1 + \frac{4k_1k_3f_c'}{\alpha p E_s K\epsilon_u}} \right] \qquad (3)$$

Eq. (3) includes two unknown quantities: $K\epsilon_u$ and k_1k_3. Studies of the measured steel stresses at failure indicated that $K\epsilon_u$ is primarily a function of the a/d ratio and k_1k_3 is a function of the concrete strength f_c'. The following empirical equations were developed.

$$10^4 K\epsilon_u = \frac{1.116\frac{a}{d} + 0.174}{\frac{a}{d} - 0.872} = 3\frac{a}{d} \text{ (always larger than zero)}$$

$$k_1k_3 = \frac{800 + f_c'}{70 + f_c'}$$

*Compatibility: $k_u(K\epsilon_u + \epsilon_s) = K\epsilon_u$
Equilibrium: $k_1k_3f_c'k_udb = N + pbdf_s$

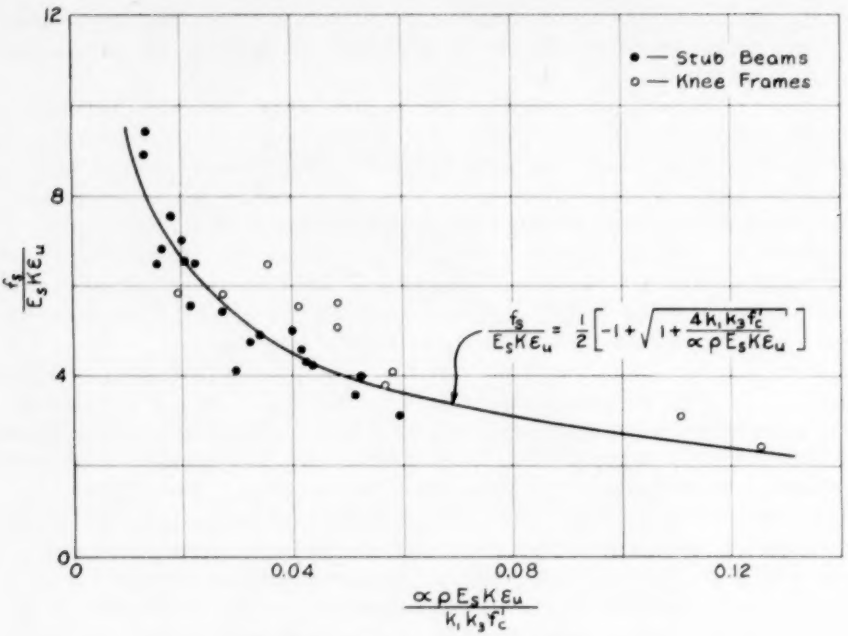

Fig. 13—Steel stress at shear failure

According to Eq. (3), the measured steel stresses for all specimens failing in shear compression may be plotted in one graph as is shown in Fig. 13. Although considerable scatter is present, the correlation in Fig. 13 is in general agreement with the curve representing Eq. (3). The major discrepancy between Eq. (3) and the test data was observed for specimens with very low percentages of reinforcement; for such specimens Eq. (3) yields consistently low values of the steel stress.

Eq. (2) and (3) contain parameter α. This parameter is equal to 1 for stub beams, but for knee frames varies with the unknown quantities N and f_s. When V/N is known, a fast solution by successive approximations may be obtained if the parameter α is expressed as*

$$\alpha = 1 + \frac{j_u}{\frac{V}{N}\frac{a}{d} - \frac{h}{2d} + \frac{y}{d} + 1 - j_u}$$

where

$$j_u = 1 - \frac{k_2}{k_1 k_3}\frac{\alpha p f_s}{f_c'}$$

*Based on the equilibrium equation: $Va - N(h/2 - y - 1 + j_u) = Tj_ud$

TABLE 8—MOMENT CAPACITY AT SHEAR FAILURE*

Specimen	Moment at failure $\frac{M_u \text{ test}}{f_c' bd^2}$	Moment at failure $\frac{M_u \text{ calc.}}{f_c' bd^2}$	Ratio $\frac{M_u \text{ test}}{M_u \text{ calc.}}$	Specimen	Moment at failure $\frac{M_u \text{ test}}{f_c' bd^2}$	Moment at failure $\frac{M_u \text{ calc.}}{f_c' bd^2}$	Ratio $\frac{M_u \text{ test}}{M_u \text{ calc.}}$
Shear failure before yielding of reinforcement							
Knee frames				**Stub beams**			
F21B2	0.352	0.352	1.00	B14B2	0.216	0.242	0.89
B2R	0.348	0.308	1.13	E2	0.182	0.160	1.14
B4	0.215	0.222	0.97	A4	0.202	0.221	0.91
B4R	0.242	0.228	1.06	B4	0.164	0.180	0.91
C4R	0.243	0.200	1.22	E4	0.152	0.146	1.04
D4	0.245	0.183	1.34				
				A6	0.184	0.190	0.97
A6	0.224	0.196	1.14	B6	0.143	0.136	1.05
B6	0.240	0.185	1.30				
				B21B2	0.224	0.254	0.88
F38A6	0.153	0.154	0.99				
B6	0.125	0.136	0.92	A4	0.226	0.195	1.16
				B4	0.189	0.182	1.04
		Average	1.107				
		Standard deviation	0.137	A6	0.177	0.189	0.94
				B6	0.158	0.143	1.11
				B28B2	0.242	0.216	1.12
				E2	0.157	0.139	1.13
				A4	0.202	0.177	1.12
				B4	0.133	0.144	0.92
				E4	0.139	0.121	1.15
				A6	0.131	0.162	0.81
				B6	0.127	0.124	1.02
				B40B4	0.110	0.125	0.88
						Average	1.010
						Standard deviation	0.109
		All specimens: Average	1.043				
		Standard deviation	0.126				
Shear failure after yielding of reinforcement							
Knee frames				**Stub beams**			
F21E4	0.214	0.194	1.10	B21E2	0.233	0.210	1.11
F4	0.199	0.183	1.09	E4	0.229	0.198	1.16
				E4R	0.176	0.150	1.17
F38E2	0.181	0.178	1.02	F4	0.191	0.157	1.22
F55E4	0.160	0.166	0.96	B56E2	0.186	0.167	1.11
		Average	1.043			Average	1.154
		All specimens: Average	1.104				
		Standard deviation	0.074				

*For knee frames, moments are taken with respect to steel at the section of the knee corner; for stub beams moments are taken at face of column stub. Dead load is neglected.

Estimating the value of j_u, evaluating α, f_s, and j_u, then repeating the operations with the computed j_u usually leads to the correct answer in two or three cycles.

With k_2/k_1k_3, and f_s determined as discussed above, moment M_{sc} at shear compression failure may be calculated from Eq. (2). Values thus computed for all knee frames and stub beams failing in shear are compared with the test data in Table 8. The test data in Table 8 include only the effects of

applied loads since the dead load moments were estimated to account for less than 2 percent of the total. The data are divided into two groups. The first group includes all specimens which failed before yielding of the reinforcement; for these, the steel stress was computed from Eq. (3). The second group includes all specimens which failed after the reinforcement had yielded; for these the steel stress was taken as the yield point value.

In the group of failures before yielding of reinforcement, the average ratio of test to computed values is higher for the knee frames than for the stub beams. On the other hand, in the group of failures after yielding of reinforcement the average ratio is higher for stub beams than for knee frames. Furthermore, comparisons of identical specimens (F21B2R and B21B2, F21B4R and B21B4, F21B6 and B21B6) show similar ratios for the corresponding specimens. It may be concluded, then, that the correlation between the test data and the calculated values is about equally good for the knee frames and for the stub beams.

The analytical expressions for predicting shear moment capacity of the knee frames and of the stub beams are the same, the only difference being that for stub beams $N = 0$. Thus, the fact that these analytical expressions are equally good for both types of specimens indicates that within the limits of these tests axial compression causes no significant difference in behavior of reinforced concrete members failing in shear compression, other than that accounted for by the changed conditions of statical equilibrium.

Flexural failures

The flexural strength of reinforced concrete members subjected to a combination of moment and axial compression was investigated by Hognestad[6] who presented a set of equations for the ultimate flexural load. Since shear seems to affect strength primarily through the formation of diagonal cracks, it appears reasonable to assume that the flexural strength of reinforced concrete members subjected to a combination of moment, shear and axial load should be in good agreement with Hognestad's equations. For sections reinforced on the tension side only and failing after yielding of the steel, Hognestad's equations may be written in the following form:

$$\frac{M_{st}}{f_c' bd^2} = \frac{\alpha p f_y}{f_c'}\left(1 - \frac{0.55}{k_3}\frac{\alpha p f_y}{f_c'}\right) \qquad (4)$$

where f_y is the yield point of the reinforcement and k_3 is the ratio of the flexural compressive strength to the cylinder strength f_c'.* For horizontally cast specimens $k_3 = 1.0$ has been suggested;[7] thus Eq. (4) differs from Eq. (2) only in that f_y is substituted for f_s and $k_2/k_1k_3 = 0.55$ instead of 0.44. As in Eq. (2), α is equal to 1 for beams ($N = 0$); for knee frames it may be evaluated by a procedure similar to that already described.†

*Hognestad used $k_3 = 0.85$; his specimens were cast vertically.
†Using $f_s = f_y$.

The values calculated from Eq. (4) are listed in Table 9 together with the test values and with the ratios of test to calculated values. The test values were computed from the applied loads neglecting the effect of dead load; the estimated dead load moments were less than 2 percent of the total moments except for the two longest specimens for which dead load moments were equal to about 5 percent of the total.

TABLE 9—MOMENT AT FLEXURAL FAILURE*

Specimen	Moment at failure		Ratio
	$\frac{M_{st}\text{ test}}{f_c' bd^2}$	$\frac{M_{st}\text{ calc.}}{f_c' bd^2}$	$\frac{M_{st}\text{ test}}{M_{st}\text{ calc.}}$
Knee frames			
F21E2	0.248	0.204	1.21
G4	0.124	0.096	1.29
F55E2	0.218	0.186	1.17
F113B4	0.272	0.250	1.09
Stub beams			
B21G4†	0.141	0.083	1.70
B113B4R	0.204	0.196	1.04

*For knee frames, moments are taken with respect to steel at the section of knee corner; for stub beams, moments are taken at the face of the column stub. Dead load is neglected.

†Diagonal tension crack formed before yielding; diagonal tension crack did not form in the remaining specimens.

It can be seen in Table 9 that test values were considerably larger than computed ones. The differences may have been caused by strain hardening and perhaps also by improper functioning of the supports.

In beam B21G4 a diagonal tension crack formed before yielding. Nevertheless, its failure was classified as flexural because of extremely large deformations prior to failure. Because of the presence of the crack, the moment at failure of this beam was computed from Eq. (2) with $f_s = f_y$.

The load at first yielding of B21G4 was a few percent higher than the computed value. Calculations have indicated that failure should have occurred at a load only slightly higher than the load at first yielding. In reality, however, the load increased almost 60 percent after the first yielding of the steel. The explanation of this unexpected increase appears to be in the combined effects of strain hardening and improper functioning of the supports.

Comparisons in Table 9 show clearly that Hognestad's equations yield safe values for the flexural capacity of knee frames and stub beams.

Modes of failure

Eq. (1), (2), and (4) for diagonal tension cracking, shear compression, and flexural failures, respectively, make it possible to predict the mode of failure of knee frames and stub beams. The relationship between the various modes of failure is illustrated in Fig. 14 for knee frames of rectangular cross section 12 in. wide and 16 in. deep, with effective depth of 14.5 in., concrete strength of 4000 psi, and yield point of reinforcement of 60,000 psi.

In Fig. 14a the load P at failure is plotted as a function of the percentage p of the longitudinal reinforcement for knee frames with a shear span of 21.6 in. Four curves are included, each representing various modes of failure or diagonal tension cracking. It can be seen that frames with p smaller than 0.46 percent should fail in flexure at a load lower than that required for the formation of a diagonal tension crack. In frames with p = 0.46-0.50 percent a diagonal tension crack forms first and failure follows at a higher

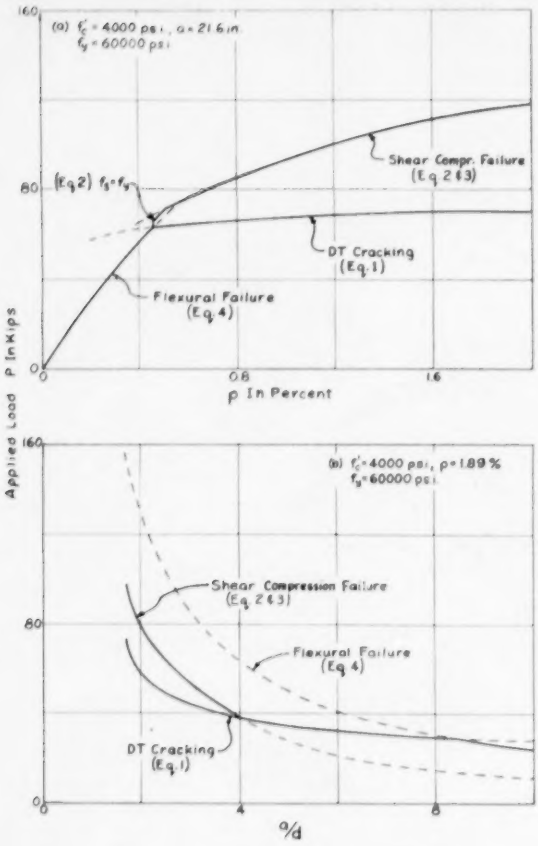

Fig. 14—Theoretical modes of failure—knee frames

load; yielding of the reinforcement precedes failure and the mode of final failure may be classified as either flexural or shear, depending on the magnitude of deformations. Finally, in frames with p larger than 0.5 percent, a diagonal tension crack forms first and is followed at a higher load by shear compression failure before yielding of reinforcement.

The sequence of modes of failure illustrated in Fig. 14a holds only for frames with short shear spans. The dependency of the mode of failure on the length of the shear span is illustrated in Fig. 14b in which the applied load P is plotted as a function of the a/d ratio for knee frames with $p = 1.89$ percent. Curves representing diagonal tension cracking, shear compression, and flexural failures are included. It can be seen that in frames with the a/d ratio smaller than 4.0 a diagonal tension crack forms first and shear compression failure follows at a higher load. In frames with $a/d = 4.0$-8.3 the diagonal tension crack formation constitutes failure (diagonal tension failure) because in this range shear compression strength is lower than diagonal

tension strength. Finally, in frames with a/d larger than 8.3 flexural failure takes place before the formation of a diagonal tension crack.

All modes of failure described above were observed in this investigation as has been pointed out in the discussion of Table 6. The observed modes of failure permit a further check of the analytical expressions; such a check was made for each specimen as follows. *For each specimen* flexural and diagonal tension strengths were computed from Eq. (4) and (1). If flexural strength was smaller than diagonal tension strength, the specimen should have failed in flexure. If diagonal tension strength was smaller, the shear moment capacity was computed from Eq. (2) with steel stresses as given by Eq. (3), but not larger than the yield point value. If diagonal tension strength came out larger than shear compression strength, the specimen should have failed in diagonal tension. If shear compression strength was higher, failure should have been in shear compression or, if considerable yielding was possible, failure may have been designated as flexural. For 64 specimens including 36 shear compression failures, 23 diagonal tension failures, and five flexural failures the computed mode agreed with the observed one; for six specimens the computed mode differed from the actual.

Specimens with incorrectly predicted mode of failure are listed in Table 10. It may be noted that all but two specimens listed had a low percentage of reinforcement. For the two exceptions, F38B4 and B113B4, ultimate loads for the expected and actual modes of failure were only a few percent apart

TABLE 10—COMPARISONS OF PREDICTED AND ACTUAL MODES OF FAILURE*

Specimen	Calculated load, kips			Mode of failure†		Ultimate load	
	Flexural failure	Diagonal tension cracking	Shear failure	Predicted	Observed	test, kips	$\frac{\text{test}}{\text{calc.}}$
Specimens with incorrectly predicted mode of failure							
F21E2‡	47.2	45.2	40.5	D	F	55.5	1.23
F38E2‡	28.3	27.5	21.8	D	S	29.2	1.05
F38B4	95.3	54.6	57.9	S	D	55.0	0.95
F55E4‡	42.5	39.9	28.7	D	S	41.4	1.04
B56E2	31.6	32.5	21.4	F	S	35.8	1.13
B113B4	47.3	49.4	23.6	F	D	46.9	0.99
Specimens with correctly predicted mode of failure but incorrectly predicted steel stress							
F21E4‡	114.5	77.9	97.8	S	S	129.0	1.32
F21F4‡	98.8	70.3	84.7	S	S	109.0	1.29
B21E2‡	83.0	46.2	72.8	S	S	95.0	1.30
B21E4‡	160.1	72.0	134.2	S	S	190.0	1.41
B21E4R‡	164.2	81.8	158.5	S	S	195.0	1.23
B21F4‡	170.1	80.9	156.5	S	S	210.0	1.34

*Except for the first six specimens listed in this table, the predicted mode of failure agreed with the observed mode.
†S = shear failure; F = flexural tension failure; D = diagonal tension failure.
‡Yielding of steel observed in the test, but Eq. (3) gave stresses lower than the yield point value.

so that the discrepancy between predicted and observed modes was probably a matter of chance.

It has been pointed out in discussion of the steel stresses at shear compression failure that Eq. (3) yields too low values for beams with low percentages of reinforcement. The discrepancies between predicted and observed modes of failure for F38E2, F55E4, and B56E2 were a direct cause of this inaccuracy. Furthermore, for six additional specimens, also listed in Table 10, Eq. (3) predicted elastic steel stresses at shear compression failure, but actually the shear compression failure occurred after some yielding had taken place. This serves to illustrate further the inaccuracy of Eq. (3) for specimens with low percentages of steel.

CONCLUDING REMARKS

Tests reported in this paper have shown conclusively that reinforced concrete members without web reinforcement and with moderately long shear spans fail upon formation of the first diagonal tension crack. Such failures are sudden and complete. On the other hand, in members with short shear span one or more diagonal tension cracks form first and a shear compression failure follows at a higher load. However, even in short beams the presence of a diagonal tension crack is dangerous, because diagonal tension cracks in beams without web reinforcement are considerably wider than ordinary

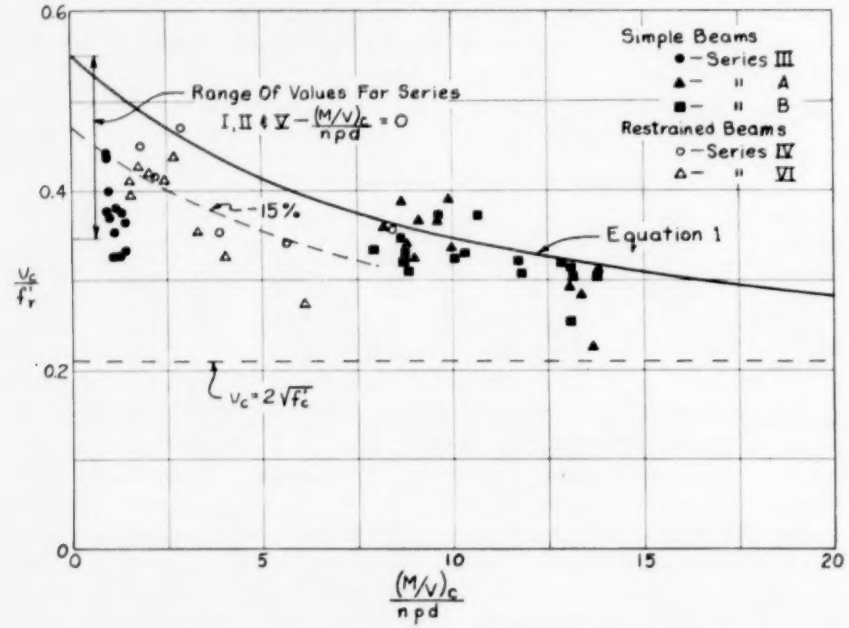

Fig. 15—Diagonal tension cracking in simple and restrained beams without web reinforcement

tension cracks; furthermore, a few repetitions of load may cause the diagonal tension crack to spread and possibly result in splitting along the reinforcement or in a premature shear failure. Accordingly, for beams without web reinforcement the diagonal tension cracking load may have to be considered in the design as the ultimate capacity.

A check on applicability of Eq. (1) for the diagonal tension cracking to structural members other than knee frames and stub beams is presented in Fig. 15 including test data for prismatic simple and restrained beams.[8,9] Simple beams of series A and B failed either at or slightly above the diagonal tension cracking load; they are in good agreement with Eq. (1). On the other hand, simple beams of series III and all restrained beams failed at loads in excess, usually considerably in excess, of the diagonal tension cracking load; these data fall consistently below the line of Eq. (1). It is believed that a part of this discrepancy was caused by differences in the methods of evaluating diagonal tension cracking loads: had the diagonal tension cracking loads for beams of series III, IV, and VI been evaluated by the same criterion as that used in this investigation, the loads would have been 10 to 20 percent higher. The differences in the criteria for diagonal tension cracking become meaningless if failure occurs simultaneously with formation of the first diagonal tension crack. Thus to resolve the question of correlating diagonal tension cracking strength of restrained and simple beams, it is desirable and necessary to compare test data on diagonal tension cracking of long beams. No such data are available for restrained beams.

It should be noted in studying Fig. 15 that test data for restrained beams follow the trend of Eq. (1). It may be concluded then that the quantities chosen as ordinates and abscissas in Fig. 10 and 15 are probably the proper parameters for correlating the diagonal tension cracking load.

It is also interesting to note that the curve representing Eq. (1) is asymptotic to $v_c/f_r' = 0.15$ and that within the practical range v_c is always larger than $2\sqrt{f_c'}$. However, it should be borne in mind that all data included in Fig. 15 are for rectangular structural members loaded with concentrated

TABLE 11—COMPARISON WITH ALLOWABLE SHEARING STRESSES

Specimen	v_c,* psi	$\frac{v_c}{f_r'}$	v_{allow},† psi	$\frac{v_c}{v_{allow}}$	Steel stress,* ksi	Minimum pf_s/f_r'‡
B56B2	148	0.070	64	2.32	27.4	0.35
A4	200	0.055	90	2.22	29.7	0.27
B4	181	0.046	90	2.01	33.9	0.19
E4	161	0.039	90	1.79	49.0	0.15
A6	269	0.047	90	2.98	32.5	0.26
B6	202	0.031	90	2.24	36.7	0.11
B70B2	132	0.056	71	1.86	37.2	0.31
A4	195	0.049	90	2.16	37.2	0.25
A6	272	0.042	90	3.02	35.0	0.23
B84B4	166	0.042	90	1.84	50.6	0.26
B113B4	155	0.033	90	1.71	yield	0.27

*Test data at failure load.
†Allowable value, ACI 318-56.
‡Computed on the basis of f_s = 40,000 psi except for B56E4, B84B4, and B113B4 in which steel stress exceeded 40,000 psi; f_s = 50,000, 55,000, and 68,000 psi were used for the three specimens, respectively.

TABLE 12—COMPARISONS WITH MOODY'S EQUATION FOR PRISMATIC BEAMS

Specimen	$\frac{M_u \text{ calc.}^*}{f_c' bd^2}$	$\frac{M_u \text{ test}}{M_u \text{ calc.}}$
B14B2	0.182	1.19
E2	0.138	1.32
A4	0.175	1.15
B4	0.150	1.09
E4	0.126	1.21
A6	0.143	1.29
B6	0.111	1.29
B21B2	0.193	1.16
A4	0.156	1.45
B4	0.129	1.46
A6	0.143	1.24
B6	0.112	1.41
B28B2	0.189	1.28
E2	0.134	1.12
A4	0.156	1.29
B4	0.136	0.98
E4	0.115	1.21
A6	0.140	0.94
B6	0.116	1.10
B40B4	0.128	0.86

*Computed from Eq. (3c), (5), and (6b) in reference 1.

loads. Studies of some existing test data[10,11] indicate that the diagonal tension cracking strength of T-beams is higher than that of rectangular beams. Furthermore, recent tests[12] of rectangular beams with uniform loading indicate an increase of diagonal tension cracking strength as compared with beams subjected to one or two concentrated loads. On the other hand, it appears reasonable to expect that direct tensile stresses may lower diagonal tension strength.

It may be noted in Table 6 that the lowest values of nominal shearing stress at failure were observed in stub beams failing in diagonal tension. The nominal shearing stresses at diagonal tension cracking, *i.e.* at failure, of all such beams are compared in Table 11 with the allowable nominal shearing stresses. In addition to the nominal shearing stresses, the table includes the steel stress at failure and the reinforcing index pf_y/f_c'. In all beams, except B113B4, steel stress at failure was below the yield point value, and in most beams it was below the minimum yield point required for intermediate grade steel. The reinforcing index, computed as described in the table, indicates that the reinforcement percentage would have been acceptable according to criteria for the ultimate strength design[13], but in all but three beams substantially exceeded the ACI 318-56[14] balanced percentage.*

The ratios v_c/f_c' in Table 11 show a decrease with increasing concrete strength; for concretes with $f_c' = 4000$ psi, the value of this ratio *at failure* varied from 0.031 to 0.047 as compared with the ACI 318-51[15] *allowable* value of 0.03. The current ACI 318-56[14] imposes an upper limit of 90 psi on the allowable nominal shearing stress; the allowable values computed on that basis are listed in Table 11 as v_{allow}. The ratio v_c/v_{allow} indicates the factor of safety against diagonal tension failures; it can be seen that for all but two beams listed this factor of safety was less than 2.5 and for four beams was less than 2.0. Considering the brittle nature of diagonal tension failures, it may be concluded that the ACI 318-56 provisions for shear in beams without web reinforcement did not provide an entirely adequate factor of safety for several of the beams listed in the Table 11; nevertheless, these provisions represent a definite improvement as compared with ACI 318-51.

*The reinforcing index for the ACI 318-56 balanced design and allowable steel stress of 20,000 psi is 0.18. The maximum reinforcing index permitted by the ACI-ASCE joint committee report on ultimate design is 0.40.

Since it appears that reinforced concrete members without web reinforcement should be designed with ultimate load equal to diagonal tension cracking load, the shear strength of such members is of a secondary importance. However, earlier studies have indicated that knowledge of the shear strength of beams without web reinforcement may be utilized in evaluating the strength of beams with web reinforcement.[1,2,3]

Earlier studies of prismatic beams[1,2,3] have shown that the shear moment capacity of beams without web reinforcement is approximately independent of the length of shear span. On the contrary, it was found in this investigation that the shear moment capacity of stub beams and of knee frames increases with decreasing shear span. It is possible that this apparent discrepancy is the result of the presence of the re-entrant corner at the compression surface of the critical section. If this is the case, the shear moment capacity of knee frames and stub beams must be always in excess of that for prismatic members.

The moments at shear compression failure of stub beams calculated as for prismatic members[1] are compared in Table 12 with the test data. It can be seen that, with two exceptions, the shear moment for stub beams was consistently higher than that predicted for comparable prismatic beams. This finding not only suggests that the presence of the re-entrant corner at the critical section may have increased the shear compression strength of short members, but also shows that it is safe to use the prismatic beam shear moment equations for predicting the shear compression strength of frame members.

SUMMARY

Seventy-one reinforced concrete specimens of rectangular cross section were tested in order to determine the shear strength of members without web reinforcement subjected to combined moment, shear, and axial compression. Thirty-three of these specimens were knee frames having the axial compression equal to the shear, and 38 were stub beams subjected to moment and shear only. The major variables were length of shear span, strength of concrete, and percentage of reinforcement.

All specimens were tested with static loads applied in several increments to failure. Of the 71 specimens, 39 failed in shear compression, 25 in diagonal tension, and six in flexure; the results for one specimen were judged unreliable. The shear compression failures occurred in the shortest specimens at loads higher than those causing the formation of diagonal tension cracks; the diagonal failures occurred in specimens with medium long shear spans; the flexural failures occurred in very long specimens and in shorter specimens with very low percentages of reinforcement. All shear compression failures and especially diagonal tension failures occurred very suddenly whereas flexural failures occurred slowly after large deflections had taken place.

Analytical expressions were developed for predicting the diagonal tension cracking and shear compression strengths. The diagonal tension cracking

equation is based on the concept of principal tensile stresses; it correlates the test results in terms of the nominal shearing stress at diagonal tension cracking. Comparisons of this equation with test data other than those obtained in this investigation, and the equation itself indicate that if the nominal shearing stress at ultimate load does not exceed $2\sqrt{f_c'}$, the strength of a structural member without web reinforcement ordinarily will not be governed by diagonal tension.

The equation for the shear compression strength is basically the same as those derived earlier for prismatic simple and restrained beams. Some differences in several empirical factors indicate that the shear compression strength of short members with re-entrant corner at the critical section may be somewhat higher than for prismatic beams.

The flexural tension strength of members subjected to a combination of moment, shear, and axial compression, or to a combination of moment and shear may be computed from the well known equations for eccentrically loaded columns and for beams.

Results of this investigation have shown that *within the limits of this investigation* axial compression affected the shear and diagonal tension strengths only insofar as it changed the conditions of statics.

Both shear and diagonal tension failures were observed after yielding of the longitudinal reinforcement. It appears, therefore, that shear failures may occur during the formation of plastic hinges. However, it must be noted that in none of the specimens in this investigation true plastic hinges were present since fairly large increases of moment resistance were always observed after first yielding of the longitudinal reinforcement.

ACKNOWLEDGMENTS

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Credit for initiating and planning of the program belongs to the joint ACI-ASCE committee on shear and diagonal tension under the chairmanship of C. S. Whitney, and to Eivind Hognestad, formerly research associate professor of theoretical and applied mechanics, University of Illinois.

Bengt Broms, formerly research assistant, A. C. Bianchini, instructor, B. R. Juskie, formerly research assistant, and M. R. Penny, junior laboratory mechanic—all of the department staff—participated in various phases of the test program.

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Title No. 53-48

Structural Refractory Concrete*

By HERMAN G. PROTZE†

SYNOPSIS

Theoretical and practical problems in the construction of jet engine test cell exhaust structures are considered, including the development and use of proper materials, mixtures, equipment, and methods for installation of durable structural refractory lightweight aluminous cement concretes. The author draws from laboratory and field experiences on four projects in recommending current optimum techniques for such work.

INTRODUCTION

Development of the jet aircraft engine is changing not only the future of aviation, but also is affecting many tangential fields, including the concrete industry. Aircraft engines require testing during development and production. Such tests are normally conducted within concrete "test cells," comprising an explosion-safe room to house the engine, a control room with appurtenant physical equipment for measuring engine characteristics, and an exhaust stack which serves to attenuate the engine roar and direct the exhaust gases skyward. Since the jet engines discharge gases which can raise the temperature within the exhaust stack from room temperature to 1200 F in 8 sec, special refractory concretes are needed to withstand both the temperature shock and the cumulative deteriorating effect of the cyclical attack of 100 to a possible 5000 test runs per year.

The structural design of the exhaust unit is not covered in this paper. The most satisfactory installations to date are L-shaped structures of rectangular or circular cross section 10 to 25 ft in diameter, with horizontal runs of 20 to 25 ft and vertical rises of 25 to 65 ft. The structures have been cast in place, precast in box-like sections, and even precast into small unreinforced panels with ship-lapped joints similar to silo staves. Wall thicknesses normally vary from 8 to 12 in.

Although industry had wide experience with high temperature cements and aggregates in refractory concretes for nonstructural uses such as chimney linings, kiln linings, and car tops, such experience was not directly applicable to this problem. Use of structural refractory concrete in jet engine exhaust

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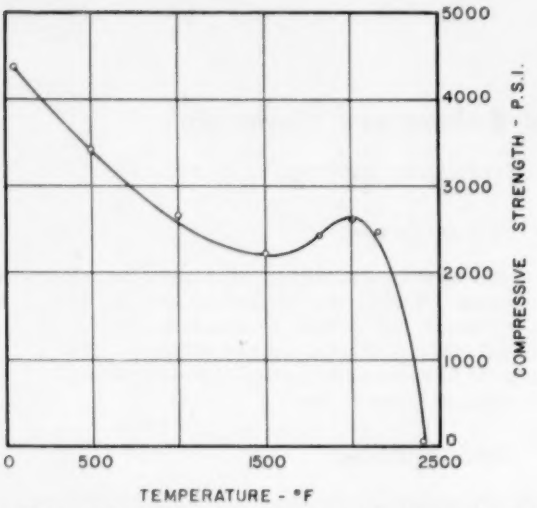

Fig. 1 — Variation in final strength of properly cured refractory concrete made with British aluminous cement and expanded shale aggregate when exposed to indicated elevated temperatures*

*Mixture 1:2.4:3.0 by dry loose volumes at total water-cement ratio of 8.0 gal. per bag, medium dry consistency. 70 F fog room curing for 7 days followed by drying at 130 F and subsequent high temperature attack on 4 x 8-in. test cylinders. Specimens tested cold after three cycles to specific temperatures shown. Additional cycles produce lower strengths.

structures has involved investigation of many variables, and the following report summarizes what are currently considered the most successful materials and best practices for construction of durable exhaust stack structures.

MATERIALS

Cement

Aluminous cement or calcium-aluminate cement as used in jet engine test cell construction in this country comes from two sources, one American and one British, at a cost of about three times that of portland cement.

The effective refractory ability of all aluminous cement is limited to certain temperature ranges. At relatively low temperatures (below approximately 300 F) dehydration and strength loss are relatively unimportant. At temperatures over approximately 1900 F ceramic bond helps to increase strength. Unfortunately the use between 950 and 1200 F is in the critical zone. A typical strength-temperature curve is shown in Fig. 1. In spite of this limitation, the use of high quality aluminous cement concrete is still desirable because in practice the temperature attack and consequent weakening effect upon the concrete by the hot gases is actually slight inasmuch as the attack occurs from one side only and penetration is to a small depth (a matter of several inches).

Typical analyses of aluminous cements are given in Table 1. Recommended limiting chemical and physical specifications for such refractory cement are also given. Inasmuch as the American Society for Testing Materials C 109-50† cube tests on standard mortars made with refractory cements are not truly

†Superseded by ASTM C 109-54T since preparation of the original manuscript.—Editor.

indicative of the relative qualities of cements in the finished concrete, a compressive strength test of concrete is recommended, using the specific materials, methods, and hydration temperatures contemplated in the work.

Supplementary characteristics of the cement such as hydration rate, hydration temperatures, strength of cores taken from large prototype sections, and effect upon strength by controlled temperature attack are also important in selecting a cement.

TABLE 1—ANALYSIS OF ALUMINOUS CEMENTS*

Cement constituents	Typical cement analysis, percent		
	American	British	Recommended specifications
SiO_2	9.6	5.3	4.5 to 11
$Al_2O_3 + TiO_2$	41.1	39.2	37 to 44
Fe_2O_3	4.9	12.2	4 to 13
FeO	5.6	4.5	4 to 7
CaO	36.8	38.0	36 to 39
MgO	1.0	0.9	2.0 maximum
SO_3	0.2	0.1	0.5 maximum
Insoluble residue	0.8	0.4	2.0 maximum
Additions	Some	0.0	0.5 maximum
Other properties	**Typical American**	**Typical British**	**Recommended**
Normal consistency, percent	24.5	23.5	No limit
Initial set, hr and min	11:15	8:25	1 to 14
Autoclave volume change, percent	−0.02	−0.04	—
325 mesh sieve, percent	82		—
Wagner S. A.	1370	1500	—
Compressive strength (C 109)			
1 day, psi	4550	5465	—†
7 days, psi	5630	7500	—†
28 days, psi	6520	8190	—†
Compressive strength (concrete)			
24 hr, psi	4850	6210	3450+
28 days, psi	6225	10875	5750+

*Physical tests are conducted in conformity with ASTM standard methods. Chemical analyses are essentially similar to ASTM methods except for refinements occasioned by the presence of special oxides indicated.

†Tests for compressive strength are described in the text under Materials—Cement. Special supplementary studies are also recommended—see Table 4 and text under Concrete—Preliminary tests.

Admixtures

In the case of aluminous cement concrete, the use of admixtures often is warranted to aid plasticity and/or to control rate of hydration and consequent temperature rise. Normally sufficient admixture is used to limit the hydration temperature of the mass to 90 F in approximately 6 hr. and to develop the final maximum hydration temperature in approximately 10 hr. Admixtures should be employed with great caution and only after thorough investigation with the given materials and existing conditions. Exhaustive check tests are required to determine the effect upon temperature rise and early attributes (such as setting time and early strength) and upon long time properties (such as final strength and retrogression). One brand of aluminous cement today already contains a small amount of interground addition which acts as both a plasticizer and retarder.

Admixtures found most valuable at times to date include the following:

1. Metallic chlorides such as $AlCl_3 \cdot 6H_2O$ in amounts not to exceed 1.0 percent by weight of cement as a control for rate of setting.
2. Air-entraining agents (sulfonated tri-ethanolamine type) in amounts up to 1 oz per bag for use as a workability agent in harsh low slump concrete.
3. Other proprietary compounds such as calcium lignosulfonate materials with dispersing and wetting agents; or protein soybean derivatives.

Fig. 2 shows typical time versus temperature rise curves for aluminous cement concretes with different amounts of admixture.

Aggregates

The most dependable and proven aggregates for use with repeated cycles

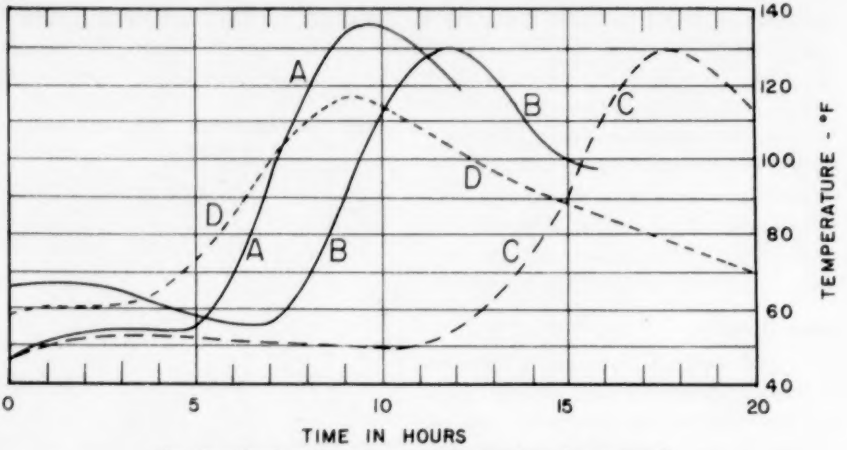

Fig. 2—Typical time versus temperature rise curves*

*All mixtures 1:2.2:2.8 by dry loose volumes of aluminous cement to Buffalo expanded shale at water-cement ratio of approximately 8.0 gal. per bag total at medium consistency. Variables as follows:

Mix number	Cement source	$AlCl_3$ Admixture	Temperature F		
			Air	Mix	Cooling water
A	British	0.4 percent	50	47	34
B	British	0.8 percent	49	66	35
C	British	0.8 percent	50	47	34
D	American	None	60	58	48

Curves A and D typical of British and American cements under reasonable conditions of cooling; note similarity of 90 F stripping time and peak temperature time. Mix B had higher original temperature than A but use of more admixture controlled temperature rise and lengthened occurrence of peak time (to limit of economy). Mix C had proper original mix temperature but use of excessive admixture (under such conditions) lengthened peak time without reducing actual peak temperature.

of high temperature shock are the artificial aggregates, manufactured by sintering or burning in a kiln certain types of clay or shale to cause expansion, aeration, and fusion. Even within the artificial aggregate family a great variety exists, and we have found that the hardest, densest, "lightweight" aggregates are superior for this use. Table 2 shows a typical comparison of a few lightweight refractory aggregates in the type and weight range considered for this work.

TABLE 2—COMPRESSIVE STRENGTH AS AFFECTED BY AGGREGATE QUALITY AND CYCLICAL HEAT TREATMENT†

Series	Aggregate	Source	Compressive strength, psi	
			Normal 70 F curing, 28 days	3 cycles at 1200 F
1	Cinders	Boston	2300	600
2	Foamed slag	Bethlehem	1700	990
3	Bloated slag	Great Lakes	2800	1600
4	Expanded shale	Ohio	4320	1950
5	Sintered shale	Pennsylvania	4600	2100
6	Fire brick grog	Pennsylvania	5400	1750
7	Expanded shale	Kansas City	5600	1820
8	Expanded shale	Buffalo	6700	2950

†All mixtures were 1:2.4:3.0 by dry loose volume, using British cement with the aggregates shown. Consistency "medium dry" (zero slump) for wet casting. Gross water-cement ratios 8.0 to 8.6 gal. per bag, including absorption. Specimens given optimum curing in fog room and tested at 28 days. Remaining specimens oven dried at 130 F, then subjected to three-cycle heat treatment to 1200 F as described under Concrete—Preliminary tests. Fifteen cycles reduces cinder concrete strength to 85 psi and Buffalo expanded shale to 2200 psi.

In selecting aggregate source a real attempt should be made to procure the highest quality aggregate available in a wide market. Aggregate should be of such quality that specimens made with the chosen aluminous cement and the *fine* aggregate under consideration should exhibit an average 7-day compressive strength of at least 90 percent of the strength of similar specimens made with the same cement and standard Ottawa sand, both mixtures to be proportioned with 1.0 parts of cement and 2.4 parts of aggregate by dry loose volumes with distilled water added to produce a flow of 100 percent. The aggregate is prewetted in this test, and except as otherwise noted, these specimens are mixed, fabricated, stored, cured, and tested in conformity with ASTM C 109-50. The *coarse* aggregate under consideration should be of such quality that 4 x 8-in. concrete cylinders made with the chosen aluminous cement plus fine and coarse aggregate from the source under consideration shall exhibit an average 7-day compressive strength at least 60 percent of the strength of 4 x 8-in. reference cylinders made with the same cement, graded Ottawa sand, and ⅜-in. trap rock, in the proportions of 1:2.0:3.0 by weight, with distilled water added to produce a 100 percent flow on the 10-in. flow table at 25 drops of ½ in. The refractory mixture is proportioned as 1:2.4:3.0 parts by dry loose volumes of cement and job aggregates, mixed to approximately a ¼-inch slump. Both mixtures are continuously fog room cured until tested.

TABLE 3—GRADATION OF REFRACTORY AGGREGATE

Sieve size	Cumulative percent retained	
	Fine	Coarse
¾ in.		0
½ in.		15–25
⅜ in.		40–60
No. 4	0	90–95
No. 8	5–15	
No. 16	35–40	
No. 30	60–75	
No. 50	75–87	
No. 100	88–93	
Fineness modulus	2.80–3.00	6.35–6.55

All sizes of aggregate should be free of inclusions and/or underburned particles which exhibit marked popping when subjected to the following test.

The aggregate is first washed, dried and regraded into separated sieve fractions; each size is then dried to constant weight and re-examined. The aggregate samples are then subjected to heat treatment as follows:

1. Place in furnace at room temperature, heat to 1200 F as rapidly as possible, and maintain temperature for 1 hr; cool for 1 hr with furnace door open; remove specimens from furnace and cool to room temperature.
2. Repeat heat treatment for three cycles.

After treatment, each fraction of each sample is regraded, reweighed, and evaluated in accordance with ASTM C88-46T. The weighted average loss for each nominal aggregate size should not exceed 5 percent.

Gradation of the refractory aggregate should satisfy the approximate compositions given in Table 3.

Water

In addition to requirements for ordinary concrete, the 28-day strength of mortar specimens made with the water under examination and chosen aluminous cement should be at least 100 percent of the strength of similar specimens made with distilled water.

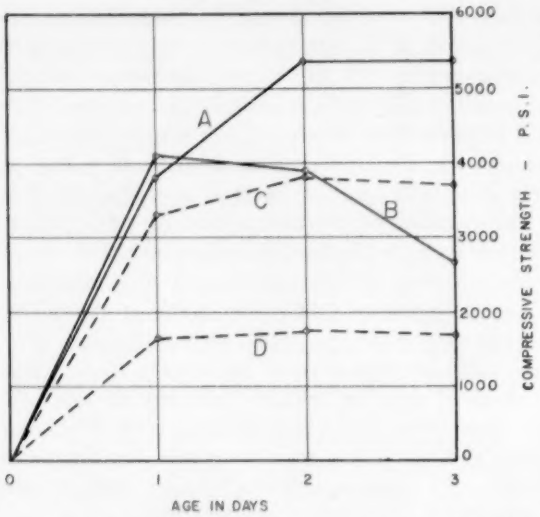

Fig. 3—Strength affected by hydration temperature (Adapted from N. Davey, reference 2)*

*All concrete mixtures 1:6 at water-cement ratio of 0.60 (by weight using British aluminous cement and river aggregates. Mixtures A and C cured with controlled cooling; mixtures B and D cured adiabatically. Original mix and maximum hydration temperatures as follows:

Mix	Mix Temperature, F	Maximum Temperature, F
A	63	63
B	63	135
C	95	95
D	95	168

Note mixtures A and C cured under 100 F exhibit reasonable strength gains. Mixture A considerably stronger than mixture C due to lower mix and curing temperature of former. Mixture B exhibits regressing later strengths relative to mix A due to uncontrolled hydration temperature (and duration). Mixture D exhibits lower strengths than mixture C due to very excessive hydration temperature (and duration). In this case retrogression undoubtedly already took place within the first day.

City tap water will normally fulfill the above requirements, except where the water is unusually "hard" or where excessive treatment has been necessary. In such cases Zeolite filtration treatment may be required, a new water source may have to be located, or the brand of cement may have to be changed, inasmuch as it has been found that different cements react entirely differently strengthwise with given sources of water.

CONCRETE

General

Proper planning, preparation, and plant layout, as well as proportioning, mixing, placing, curing, and cooling of the concrete are all as important as the use of proper constituents. The attendant problems vary somewhat with the design and size of the structure.

In general, the concrete mixture should employ a "medium" cement factor; that is one which is not so lean as to produce excessively harsh mixtures and low strengths, and—on the other hand—one which is not so rich as to produce excessive hydration temperature rise and consequent lowered strength and retrogression. Aggregates are blended to produce a generally straight line

gradation and give maximum density within the realm of mixtures used. The mixtures will of necessity be gritty, dry, and harsh but amenable to proper vibration. Admixtures will be used if needed to control hydration rate or workability.

Fig. 1 indicates how attack by heat affects concrete strength. Effect of heat of hydration *during* hardening is similar but even more markedly critical upon not only the actual strength of the concrete, but upon later retrogression, as shown in Fig. 3. We have found that concrete temperatures higher than 100 F during setting and hardening periods may lead to substantial weakening of the concrete and progressive retrogression in strength; such temperatures higher than 120 F will almost surely produce marked weakening and progressive retrogression. Temperatures higher than 100 F are therefore to be avoided whenever practicable and extreme care should be taken to avoid temperatures of 120 F or higher. These limits vary slightly with different cements.

As a corollary it is now apparent that to properly judge quality of concrete in the finished structure, it becomes necessary not only to obtain typical core strengths but also to obtain readings of typical hydration temperatures as a guide for the selection of locations from which to drill the cores.

Design criteria

Some structures built to date have had their design based on a working stress of 1500 psi. Inasmuch as effect of attack by extraneous heating can reduce strength as much as 60 percent by laboratory test, a core strength of 3750 psi has been established as the minimum permissible in the structure (prior to use). Although this appears to provide no factor of safety, it must be remembered that jet engine attack affects field concrete to only a moderate depth (as compared to full penetration of a small specimen attacked on all sides in a laboratory furnace) and consequently the effective factor of safety in the field is undoubtedly at least 1.5.

Secant modulus of elasticity at working stress has been determined by test as follows:

$$E_c = 500\, f_c' \text{ after drying but before any attack by heat}$$

$$E_c = 100\, f_c' \text{ after attack by heat}$$

The attack by heat utilized in the latter case was a complete "soaking" of the 4 x 12-in. test cylinders at 1200 F for three cycles as described below for other tests. In other words, with cyclical heat treatment there is approximately an 80 percent lowering of the modulus of elasticity of refractory concrete of the type specified, during a companion lowering in compressive strength of not more than 60 percent.

Test for linear expansion indicate an average coefficient of 0.0000041 in. per in. per deg F for the mixtures and temperature ranges here considered.

Mix specifications

The following limitations have proven satisfactory on a series of projects:

Basic mixture	Cement factor, 6.75 bags per cu yd, minimum Water-cement ratio, 8.5 gal. per bag, maximum Consistency, medium dry (zero slump)		
Approximate dry loose volumes	Aluminous cement one bag Fine aggregate 2.4 cu ft Coarse aggregate 3.0 cu ft		
Changes in mix proportions	Maximum cement factor, 7.5 bags per cu yd Ratio of cement to total uncombined loose dry volumes, 1:4.8–1:5.4 Ratio of fine to coarse aggregate by volume, 1:1.05–1:1.25		
	Age of concrete, days	Individual minimum compressive strengths, psi	
		Test cylinders	Cores from structure
Concrete quality	1	3000	—
	3	3500	3000
	7	4000	3300
	28	4500	3750
	60	No retrogression each case	

In the above specification, the water-cement ratio is expressed in terms of total water (added plus free plus absorption) for simplicity of determination with highly porous aggregates. For this type of concrete, using water content somewhat less than that for zero slump, there is no good measure for consistency and workability. The most practicable method of judging proper consistency for this work has been to attempt to fabricate a sample of the concrete into a 4 x 8-in. cylinder mold. If the first 4-in. layer of concrete will not permit penetration of a ½-in. puddling rod upon firm rodding, the mixture is too dry; if penetration is readily accomplished, the mixture is too wet; if, on the other hand, the rod will just barely penetrate the first layer at the start and, with continued rodding, will "pack-out" (like a sheep's-foot roller in soil) the consistency is optimum.

Proportions are expressed in terms of dry loose volumes to most readily maintain a constant quality of mixture with varying densities of aggregates. The indicated changes in mix proportions present the maximum permissible range in adjusting workability, consistency, strength, and other properties. Batching should of course be by weight measurement.

The indicated cylinder strengths apply to concrete sampled at the mixer to check inherent mix quality and in this case are valuable only for that purpose. The cylinders require moist curing under 73 F and protection from the sun's rays to produce results which are rational. Cylinders are 4 x 8 in. to permit easier fabrication before the mixture becomes unmanageable and also to reduce temperature rise due to hydration.

The requirements for core strength apply to mix placements having a maximum hydration temperature not in excess of 120 F. In such event, and with proper curing, it is reasonably safe to assume that no marked retrogression will take place. However, for aluminous cement concrete, attainment of any of the above specified minimum early strengths does not assure attainment of the companion required final strength. To this end a strength-time curve should be developed for the particular job and conditions. If the maximum hydration temperatures are over 120 F and less than 140 F minor retrogression may take place after 28 days. If the hydration temperature is between 140 to 160 F, the 28-day core strength requirement should be established at 4500 psi to assure a minimum 60-day strength of 3750 psi. (It is unlikely that this retrogression will continue markedly with time in this temperature range.) If the maximum hydration temperature is over 160 F, the concrete definitely should be rejected unless the minimum 60-day strength is at least 4000 psi and the retrogression in actual strength between 28 and 60 days is less than 15 percent loss from the 28-day figure.

It is expected that with a good degree of quality control, the median 28-day strength of 50 sets of either specimens should be at least 50 percent higher than the minimum strength specified, and the average strength of any five consecutive sets of specimens should be at least 25 percent greater than the minimum specified. On a project involving precast sections 12 in. thick and approximately 1 cu yd in volume, the following are results of tests on approximately 1000 cu yd of concrete involving 53 series of cylinders and 61 series of cores:

	Cylinders	Cores
Maximum strength	8363 psi	7918 psi
Minimum strength	3580 psi	3336 psi
Average strength	6780 psi	5750 psi

These test results included concretes cast at excessive hydration temperatures up to 174 F (which were rejected for use). The averages for cylinders and cores are respectively 50 and 53 percent above the minimums specified.

Preliminary tests

Before formal start of concreting operations, and after basic laboratory acceptance of constituents, large scale preliminary concreting runs should be carried out in the field for a number of purposes:

1. To aid in the selection of a specific brand of cement and perhaps admixture most suited to existing field conditions.
2. To acquaint the contractor first hand with the specific techniques and equipment required for the work.
3. To establish minimum control tolerances for the formal work which will be considered acceptable to produce results compatible with the specifications.

The preliminary tests should employ the basic mixture specified above. Mold nine 4 x 8-in. test specimens from this concrete and store at 70 F successively as follows: (1) in molds until concrete reaches 90 F; (2) in water for remainder of 24-hr period; (3) in fog room until tested. Three specimens

TABLE 4—EVALUATION OF CEMENT QUALITY*

Item	Cement A	Cement B
General description		
Strength potential	Moderate	High
Temperature rise	Moderate	High
Mass concrete quality†		
Mix temperature	58 F	60 F
Obtained maximum hydration temperature	115 F	126 F
Actual compressive strength, 7 days		
Cylinders	3170 psi	6330 psi
Cores	3025 psi	5280 psi
Evaluation of strength quality		
Laboratory concrete strength, 24 hr‡	6230 psi	10900 psi
Correction for field aggregate§	60 percent	60 percent
Estimated field concrete strength	3740 psi	6540 psi
Expected loss due to over 110 F	420 psi	1330 psi
Core strength after mass hydration	3320 psi	5210 psi
Loss in heat treatment (3 cycles)§	55 percent	60 percent
Relative final strength	1500 psi	2100 psi

*Moderate strength moderate heat cement of type A recommended for use where moderate strengths are specified and/or where cooling facilities are poor. Cement B recommended for use where higher strengths are required and/or where superior cooling facilities are available. For average strength, average cooling, either cement may be considered.

†Item 2 represents average actual data from 12 cylinders and 12 cores in four series of field mixtures as described in Concrete—Preliminary tests using 1:2.2:2.8 mixtures by dry loose volumes at water-cement ratio of 6.5 gal. per bag total, and expanded shale plus two different cements shown. Note relative core strengths due to differences in abilities of the cements and effect of hydration temperatures.

‡Based on laboratory concrete acceptance tests as described in text under Materials—Cement; reduced by anticipated losses due to correction for field aggregate and loss due to excessive hydration temperature, the anticipated core strength after mass hydration is obtained, similar to actual core strength shown above. Losses due to heat attack during three-cycle treatment bring results down to final relative strength.

§Based on averages of over 25 tests each.

each should be tested in compression in the moist condition at the ages of 24 hr, and 7 and 28 days. Three rounds of tests should be made, each on different days, so that the grand average strength value for each variable at each test age will represent a total of nine specimens.

A single 4 x 7-ft concrete prism, of thickness equal to the proposed field construction, should be fabricated from the described mixture on each of three separate dates. Concrete should be mixed, placed in the metal form by means of internal vibration, and cured similar to the methods described below for field casting. Accurate temperature records of constituents, air, mixture, and center of the mass should be determined, preferably by means of recording thermometers and should cover a 24-hr period immediately after casting.

At least three nominal 4 in. diameter cores should be drilled from the prisms for their *full thickness* for the determination of the variation in compressive strengths with age, at 3, 7, and 28 days after casting. The cores should be cut near the center of the panel. Cores should be stored in the fog room until tested.

From each mixture three 3 x 6-in. cylinders also should be cast for successive treatment as follows:

1. Cure 7 days in 70 F fog room.
2. Oven dry three days at 130 F.
3. Place in furnace at room temperature, heat to 1200 F for 1 hr; cool 1 hr with furnace door open; remove specimens from furnace to cool to room temperature.

4. Repeat heat treatment for a total of three cycles.
5. Cap and test in compression.

The data obtained should be compared between brands and sources of constituents to aid in selection on the basis of relative performance. Table 4 summarizes comparison tests on cements. Furthermore, the contractor may be governed by the general trends in results as to need of further refinements in mix constitution and/or cooling as described below.

Field tests

At least three test cylinders and two cores should be made representing each 25 cu yd of concrete placed, and no less than this number of cylinders and cores should be taken during one day's placement. More specimens may be required. The test cylinders and cores should be sufficient in number and of such ages, at time of test, as to establish definite trends in variation in strength with age and to determine accurately retrogression in strength with age. Whenever practicable cylinders and cores should be taken from the same batch of concrete and tested at the same ages. From each group of specimens at least one cylinder and one core should be tested at an early age (3 to 7 days); at least another cylinder and core should be tested at 28 days. Additional specimens may be tested at other ages (1 day for cylinders for control data, and occasionally at 60 days for cylinders and cores to study retrogression).

Cores should be drilled from portions of specific panels or other parts of the structure at locations where, during the hardening period, the temperature at the interior of the concrete mass was found to be the greatest.

Cores require special treatment in drilling, storage, and test for results to signify true quality of the parent concrete. They should be drilled at such a speed and properly flushed with water to avoid heating over 130 F. They should be moist cured at 70 F and stored out of the sun's rays.

Cores drilled from the structure must penetrate the entire thickness of the section. Tests on cores which do not include practically the full thickness cannot be considered as representative of the concrete in the structure, and may be misleading; such cores should be replaced by cores which penetrate the entire thickness.

Accurate records must be kept of pertinent temperatures. Thermometers or other temperature measuring devices should be calibrated at intervals to insure accurate records within 2 deg F. These records are used as a guide for corrective cooling measures and in planning subsequent installations. One thermometer shall be installed for *every 2 cu yd of concrete placed.* Ordinarily temperature readings for concrete may be made at ½-hr intervals, but to ascertain peak temperatures several readings at ¼-hr intervals may be required. The thermometers may be inserted through small holes in the forms to a depth as required to position the sensitive element at mid-thickness of the concrete or equidistant from other cooling elements (if any). Records should cover the period from placing of concrete through the time it drops

30 F beyond the peak. Dial thermometers and thermocouples have been found satisfactory for this work.

CONCRETING OPERATIONS

Materials preparation

Cement and aggregate storage piles should be kept under substantial and effective cover to protect the materials from the direct rays of the sun and allow free circulation of air between covers and the materials. All materials should be kept at as low a temperature as practicable for use, definitely under 70 F. All aggregate piles which are to be used within 2 days should be sprayed with water from fine fog sprays so that the surfaces are maintained slightly moist but not wet and the evaporation of the moisture tends to lower the temperature of the aggregate. Aggregates should be processed to a uniform state of moisture before use. A surface moisture of 1 percent has been found desirable. This moisture can be determined by oven drying test related to the known absorption. Aggregate at this free moisture content will slightly stain but not wet the hand when grasped firmly in the fist.

The temperature of the mixing, cooling, and curing water must be as low as practicable, 65 F or less, and preferably under 40 F.

Equipment cooling

Excepting when the temperature of the equipment parts, forms, etc., is 60 F or less, upon starting work for the day (or shift), and again after any major interruption in concrete placement, the mixer, portions of concrete handling equipment which comes in contact with concrete, all form surfaces, reinforcing steel, embedded conduits, tubes, and fixtures which will be in contact with concrete should be precooled to as low a temperature as is practicable. The surfaces of forms which will come in contact with concrete should be cooled immediately prior to placing concrete. In no event should form temperature exceed 70 F, and preferably 60 F.

Great care shall be taken to maintain the temperature of portions of forms which come in contact with concrete at as low a temperature as is practicable from the time the initial lot of contained concrete is placed until the forms are stripped. This temperature should preferably be under 40 F. This may be accomplished by the application of *flowing* cooling water applied to all portions of the exterior surfaces of the forms on both sides of the concrete section. Special care should be exercised to obtain complete water coverage of all surfaces.

Forms

Forms should preferably be of 3/16-in. steel. All bracing in contact with forms must be vertical to permit free flowing of cooling water downward. Lift heights should not exceed 5 ft, and preferably 4 ft.

Permanent or portable cooling sprays must be installed on each side of the forms to provide cooling of the concrete during hydration. The sprays must be designed to provide complete and uniform coverage of all surfaces. The application of heavy streams of water is neither necessary nor desirable.

Mixing

Special care should be exercised to insure that all mixing and other construction equipment is clean and free of lime, portland cement, or other materials which might affect the rate of setting of aluminous cement.

Hand mixing must not be permitted. The mixing time (for mixers of ½ cu yd capacity) should not be less than 1¼ min or longer than 1¾ min after all ingredients, including the water, are in the mixer. When the temperature of the air is less than 60 F, maximum mixing time may be increased to 2 min. Mixers should be equipped with operating timing devices. The tempering of concrete after the completion of the maximum mixing time must not be permitted.

The temperature of concrete as discharged from the mixer should be as low as practicable and definitely between 40 and 60 F regardless of weather.

Mixers should be as near as practicable to location of deposit.

Placing

Concrete in general should be placed in accordance with "Recommended Practice for Measuring, Mixing and Placing Concrete (ACI 614-42)". Panels of precast concrete should be cast as walls rather than slabs, to permit easier and more thorough cooling of maximum surface areas.

Concrete should be thoroughly compacted by means of internal vibrators. In this work low frequency (3500 rpm) high amplitude vibrators with long spuds have been found significantly more effective than the speedier modern type, valuable with more plastic mixtures containing heavier aggregates. It is important that vibrators be used in the vertical position, otherwise "pumping" of air will result, and such air will be trapped within the concrete with consequent weakening of the mass and disfigurement of the surfaces.

Cold weather requirements

Whenever the temperature of the surrounding air is below 35 F, all concrete placed should have a temperature of between 40 and 60 F, and adequate means should be provided for maintaining the temperature of air surrounding the concrete at not less than 40 F or over 60 F for at least 24 hr after placing.

Primary cooling

An important aspect of cooling is keeping the hydrating concrete reasonably cool by uniform application of flowing cold water on surfaces of forms or concrete.

Cooling water, at a temperature preferably below 40 F, should flow through the cooling sprays as described above from a time before concrete is placed until the concrete mass safely passes the peak temperature. Sometime before this period, the concrete will attain its set, and form stripping may take place.

Form removal

It has been found that ordinarily forms may be and should be removed when the temperature of the structural refractory concrete farthest removed

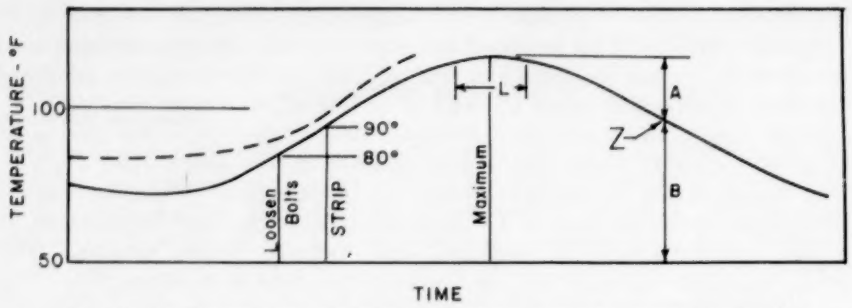

Fig. 4—Hardening characteristics of aluminous cement concrete sections*

*Solid curve represents typical time-temperature curve during first 18 hr of setting and hardening of 12-in. aluminous cement concrete sections properly mixed, cooled, and cured. Form stripping may take place at 90 F and bolts may be loosened 10 deg sooner, provided that true chemical action has begun as indicated by a rise of at least 20 F from earlier low point in curve. (Broken line indicates start of hydration curve for higher mix temperature; stripping at 90 F in this case would be too soon.) Cooling should keep maximum temperature rise *and* duration time *L* to a minimum for best quality. Formal cooling may be stopped at point *Z* and secondary curing initiated. This time is when there has been a drop in concrete temperature from the maximum of at least 30 F (*A*) or when the actual concrete temperature reaches 70 F within the mass (*B*), whichever occurs the sooner.

from cooling elements is approaching 90 F. This may be as soon as 3 or 6 hr after the concrete is placed. Forms ordinarily may be removed safely when exposed moist concrete has hardened sufficiently to withstand rubbing with a finger without soiling the finger appreciably.

Adequate provisions should be provided to remove all sections of forms with the utmost dispatch. The application of cooling water may be interrupted on either side of a concrete section while the forms are being removed from that section, provided such interruption is *less than 5 min for each side of the concrete.* While forms are being removed from one side of a concrete section, the application of cooling and/or curing water must continue on the other side of that section.

Secondary cooling or primary curing

Immediately upon the removal of any section of forms, the concrete surface which was covered by that form section must be sprayed with cooling and curing water so applied that the water flows over and completely covers all concrete surfaces. Special care should be exercised to obtain complete water coverage of all concrete surfaces and to avoid the interruption of the application of cooling and curing water. The application should be made in the same manner and to the same degree described above for form sprays. This flowing cold water should be continued at least until there has been a drop in hydration temperature of 30 F from the maximum or until that temperature reaches 70 F, whichever comes the sooner. At that time concrete sections (if precast) may be moved to other storage areas (Fig. 4).

Secondary curing

After completion of the above cooling the concrete should continue to receive uniform moist curing in a fog or spray room at a temperature not greater than 70 F until the concrete has attained an age of 48 hr. At this age the curing is complete.

Final treatment

Before aluminous concrete units or structures are put to use they require careful slow uniform drying and heating to 300 F over a period of a week to prevent rapid regression in quality during first shock by engine use.

FAILURE TO MEET REQUIREMENTS

Upon failure of either cylinder or core strengths at any age to meet minimum strength requirements, suitable corrective steps must be taken. If final strengths are low, the concrete should be rejected and future placements properly corrected.

If cylinder strengths are low, suitable correctives would include changes in source of constituents, improvement in cooling procedures, reduction of the amount of water added at the mixer, and/or increasing the cement content, within the specified limits. If the core strengths (only) are low, suitable correctives would consist essentially of reducing maximum hydration temperature by increasing the effectiveness of any or all cooling and curing procedures.

Core strengths lower than 80 percent of the corresponding cylinder strengths, lower than the minimum values specified, and/or showing retrogression in strength with age of concrete are also considered as sufficient reason for increasing the effectiveness of the cooling provisions and/or of lowering the temperature of the mix. When so indicated, such steps should be initiated without delay, and may include the following methods:

1. Lowering the temperature of the cement and aggregates.
2. Limiting the prosecution of work to cool days and nights.
3. Lowering the temperature of the mixing water by use of ice or other refrigeration.
4. Similarly lowering the temperature of the cooling water.
5. Taking steps to further minimize interruptions in the application and uniform distribution of cooling and curing water.
6. Consideration of design with thinner sections.
7. Installation of cooling tubes within the concrete sections to permit further heat removal by means of circulating cold water. This may apply to sections 12 in. thick or over.

Cooling tubes are normally thin walled tubes 1 in. O. D., placed 6 in. on center vertically at the center of wall sections. Cooling tubes, if used, should be so constructed, arranged, connected to the water supply, and

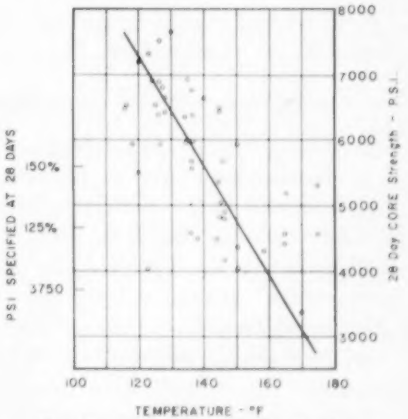

Fig. 5—Strength versus hydration temperature

Data represent approximately 1000 cu yd of concrete cast into 12 in. thick panels approximately 4 x 7 ft. Concrete 1:2.2:2.8 by dry loose volumes at total W/C ratio of approximately 8.0 gal. per bag, using British aluminous cement and Buffalo expanded shale. Horizontal scale represents *maximum* hydration temperature obtained in center of concrete mass at which location 4 x 12-in. cores were drilled during first week and stored in fog room until tested. Slope of average line: 12 F equals strength loss of 1000 psi.

drained of water to prevent damage to the concrete. Flowing water should be circulated through the cooling tubes from the time concreting is started until the entire mass has cooled to 70 F or less. After completion of the cooling period, the tubes are cut off flush with the concrete surfaces and completely filled with stiff grout composed of one part aluminous cement to two parts damp fine aggregate by volume.

Fig. 5 presents some accurate core strengths as related to hydration temperatures in panels 1 ft thick and containing approximately 1 cu yd of concrete. It is indicated that a general trend exists wherein a rise of 12 F in hydration temperature causes a lowering of the compressive strength of 1000 psi and furthermore causes troubles in retrogression. Data from another project indicated a relationship of 20 F equals 1000 psi.

CONCRETE CONTROL

This type of construction requires a detailed program for testing and control of concrete and materials. As on any job, it is essential that the inspection work be carried out in a rigid but sympathetic, intelligent manner. Specifications should preferably be written in consultation with a specialist who is acquainted with the details of the work under the field conditions obtaining, in order to produce an accurate authoritative job "guide-book" for the contractor and testing agency.

Field control is a 24-hr job and requires at least two men at all times for every 100 cu yd placed per day.

CONCLUSION

The profession has very definitely made a place for use of refractory concrete as a structural material in jet engine test cell exhaust stack construction. Such concrete is a controllable engineering material amenable to straightforward scientific direction. The material is neither treacherous nor unpredictable; to the contrary, its ultimate qualities are directly dependent upon control quality, which—if imperfect—practically assures an unsatisfactory end result!

After judicious selection of proper concreting constituents, the most important rule to follow in properly manufacturing refractory concrete is to *keep it cool*. The use of precast sections is consequently recommended for simplicity of production.

Proper planning, preparation, and plant layout are important functions of the contractor, who must have an honest will to perform.

In place cost of the described concrete has varied, in the current market, from approximately $155 to $205 per cu yd.

ACKNOWLEDGMENTS

The information presented in this paper is based upon a review of the literature, comprehensive personal research in the laboratory, and a careful

evaluation of data gathered on four major construction projects during the last 3 years. These projects include two test cells at the General Electric Co. River Works, Lynn, Mass.; 33 test cells at the General Electric Co., Lockland, Ohio; four test cells at Olmsted Air Base, Middletown, Pa.; and six test cells at Chrysler Corp., Centerline, Mich. Information from other projects was also studied.

The author wishes to thank the managements and staffs of the General Electric Realty Corp., Schenectady, and Chrysler Corp., Dodge Division, Detroit, for permission to present the data contained in this paper. The author is indebted to the Duffy Construction Co., Cleveland (Lockland project) and to the MacDonald Construction Co., St. Louis (Centerline project). The writer is also indebted to Oliver G. Julian and staff of Jackson & Moreland, Engineers, Boston; Gus Muth and staff of Albert Kahn Associated Architects and Engineers, Inc., Detroit; The Ohio River Division Laboratory, U. S. Engineer District; the John H. Black Co., Buffalo; and the Universal Atlas Cement Co., New York. The Lafarge Aluminous Cement Co., Ltd., London, has been particularly helpful to the writer in providing information and samples.

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Title No. 53-49

Design of Floating Slab Foundation*

By NORMAN B. GREEN†

SYNOPSIS

A method is developed for the stress analysis and design of a floating or monolithic type of slab foundation that supports edge loads. The method is applied to the foundation of a typical one-story wood frame school building. This type of foundation effects a considerable saving in cost over conventional concrete wall footings with a separate floor slab.

INTRODUCTION

The so-called floating slab foundation has been used quite extensively for house construction.‡ It is simply a concrete slab that is thickened and reinforced along the lines of bearing walls and partitions, so as to form a monolithic structure that serves as both floor and foundation.

This type of foundation is now being employed in Los Angeles, for one-story wood frame school classroom buildings, at a considerable saving in cost over the conventional reinforced concrete wall footings with a separate floor slab. The typical building is comprised of a series of 28 ft x 32-ft classrooms arranged end to end with an outside corridor along one side. The rafters span 28 ft, between the rear wall of the building and the front wall, which also supports the roof of the outside corridor. The front and rear bearing walls rest on opposite edges of the foundation slab.

In a modification of this plan two rows of classrooms are placed back to back with a bearing partition on the center line of the building. The foundation slab is then 56 ft wide instead of 28 ft and supports a uniform load along the center line of the slab as well as along its two edges.

COST COMPARISON

Fig. 1 is a section of a floating slab foundation for a school classroom building, showing the thickened edge under the exterior wall at the perimeter of the slab. Calculations for the slab reinforcement are contained in an example that follows. It will be noted that the bottom edge of the slab is carried down only 12 in. below grade, which is permissible with the very low soil pressures that occur under this type of foundation. For comparison the design of

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†Member American Concrete Institute, Consulting Structural Engineer, Los Angeles, Calif.

‡Christenson, C. O., "Use of Concrete in Residential Construction," ACI JOURNAL, Sept. 1952, *Proc.* V. 49, pp. 37-44.

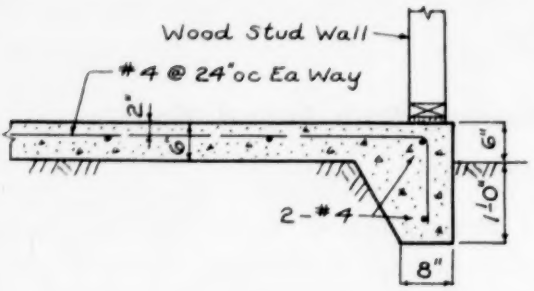

Fig. 1—Typical section of monolithic floating slab foundation

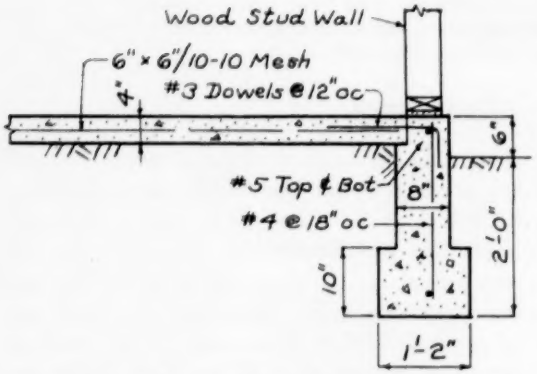

Fig. 2—Typical section of conventional wall footing and separate floor slab

Fig. 2 shows a conventional wall footing and separate floor slab that might be used for the same building. Let it be assumed that the building in each case is 28 x 96 ft in plan. Then for the design of Fig. 1 the cost of the foundation is as follows:—

Concrete 60 cu yd @ $15.00	$ 900.00
Forms 372 sq ft @ 35¢	130.00
Reinforcement 2370 lb @ 12¢	283.00
Excavation 18 cu yd @ $1.50	27.00
Total	$1340.00

and for the design of Fig. 2 the cost is:—

Concrete 50 cu yd @ $15.00	$ 750.00
Forms 1240 sq ft @ 45¢	558.00
Reinforcement 1680 lb @ 12¢	202.00
Excavation 55 cu yds @ $1.50	82.00
	$1592.00

The greater cost of the concrete and reinforcement for the floating slab foundation is more than offset by the smaller cost of the forms, so that the total cost is about 16 percent less than for the conventional design.

Use of the floating slab also saves construction time. The conventional wall footing must be placed, forms stripped after a curing period, and backfill must be placed before the separate floor slab can be cast.

SOIL PRESSURE

The slab or mat type of foundation is often employed where soil has a low bearing capacity. However the design in such cases is almost invariably based on the assumption of a uniformly distributed soil pressure, which is the only design method presented in most standard reference books on foundations. This assumption is not practicable for the type of building here considered, which is of light construction with a relatively long roof span. The foundation slab of corresponding span would have to be heavily reinforced and excessively thick, not only for strength but also in order to secure the stiffness required by the assumption of uniform soil pressure.

STRESS ANALYSIS FOR THE EDGE-LOADED FOUNDATION SLAB

A transverse section of a floating slab foundation is shown diagrammatically in Fig. 3. The wall loads P and P_1 are supported by opposite edges of the slab. Under the action of these loads the outer portions of the slab

Fig. 3—Diagrammatic section of slab foundation showing forces and deflection

AB and A_1B_1 are depressed and the resulting negative moments are resisted by upward flexure of the portions of slab BC and B_1C_1. If the foundation is sufficiently wide, a portion of it between C and C_1 will be unaffected by the two edge loads and the shear and moment in the slab between these points will be zero. Accordingly, for the purpose of stress analysis, the portion of slab AC (or A_1C_1) may be isolated.

Let Fig. 4 represent a strip of foundation slab 1 ft wide extending between A and C, with the forces acting upon it. The following notation applies to Fig. 4 and to the subsequent analysis.

P = edge load, lb per linear ft of edge
w = weight of slab, lb per linear in. of slab strip
l = length of raised portion of slab, in.
a = length of depressed portion of slab, in.
p = soil pressure at outer edge of slab, lb per linear in. of strip
e = depression of outer edge of slab, in.
V = shear in slab at point B, lb
M = moment in slab at point B, in.-lb
M_x = moment in slab at distance x from origin of coordinates, in.-lb
Δ = bending deflection of the slab between A and B with respect to the tangent at B, in.
Θ = slope of the elastic line of the slab at point B
E = modulus of elasticity of concrete
I = moment of inertia of a strip of slab 1 ft wide
m = soil constant, defined by the relation $e = p/m$

The following analysis is based on two assumptions: (1) That the soil pressure distribution under the slab is triangular with a maximum pressure

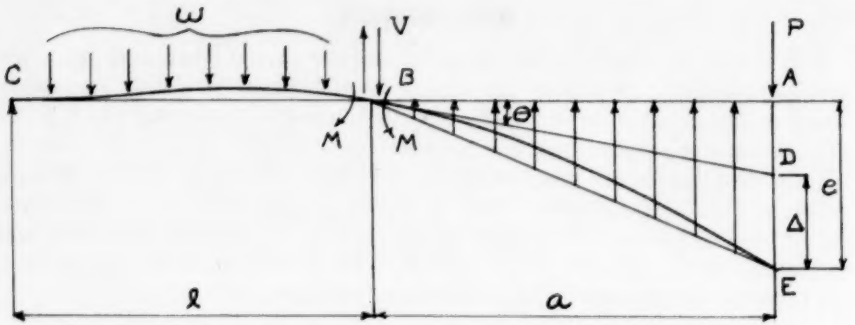

Fig. 4—Slab strip showing forces and elastic deflection

at the outer edge, as indicated in Fig. 4; (2) That the depression of the outer edge of the slab is proportional to the unit soil pressure at that point.

The first assumption is on the safe side, since the upward deflection of the slab between B and E under the action of the negative moment, as indicated in Fig. 4, will move the center of gravity of the soil pressures toward the edge A which will reduce the bending moment M in the slab. That is, in Eq. (2A) the factor 2/3 is increased.

Referring to Fig. 4 and taking moments about point C, where the moment in the slab is zero,

$$V\,l = \frac{w\,l^2}{2} + M$$

which reduces to

$$V = \frac{w\,l}{2} + \frac{M}{l} \qquad (1)$$

Consider first the portion of slab AB acted upon by downward loads P and V and an upward soil pressure varying uniformly from p at A to zero at B. In order to simplify the analysis for this portion of the slab, the term M/l in Eq. (1) will be omitted and it will be assumed that $V = wl/2$. This approximation will give a slightly larger value of M and is therefore on the safe side. Then since the pressure distribution is assumed to be triangular,

$$p = \left(\frac{w\,l}{2} + P\right)\frac{2}{a} \qquad (2)$$

taking moments about point B,

$$M = \left(\frac{w\,l}{2} + P\right)\frac{2}{3}\,a - P\,a \qquad (2A)$$

which reduces to

$$M = \frac{a\,(w\,l - P)}{3} \qquad (3)$$

The elastic equation for this portion of the slab, with origin of coordinates at B, is

$$E\,I\,\frac{d^2\,y}{d\,x^2} = \frac{p\,x^3}{6a} - \left(\frac{p\,a}{2} - P\right)x + \frac{p\,a^2}{3} - P\,a \qquad (4)$$

Integrating Eq. (4) twice there is obtained,

$$E\,I\,y = \frac{p\,x^5}{120a} - \left(\frac{p\,a}{2} - P\right)\frac{x^3}{6} + \left(\frac{p\,a^2}{3} - P\,a\right)\frac{x^2}{2} \qquad (5)$$

in which the constants of integration are zero, since both the slope and deflection are zero when x is zero. If x equals a in Eq. (5) then y equals Δ and there is obtained

$$\Delta = \frac{a^3}{E\,I}\left(\frac{11p\,a}{120} - \frac{P}{3}\right) \qquad (6)$$

Refer to Fig. 4 and consider the slope Θ of the tangent BD to the elastic line of the slab at the point B. Since $e = AE$ is intrinsically positive and Δ from Eq. (6) will have the negative sign, the distance AD is expressed as $e + \Delta$. Now if a slope downward to the right is taken as negative, there may be written,

$$\Theta = -\left(\frac{e + \Delta}{a}\right) = -\frac{e}{a} - \frac{\Delta}{a} \qquad (7)$$

In Eq. (7), substituting p/m for e and Δ from Eq. (6), there is obtained,

$$\Theta = -\frac{p}{m\,a} - \frac{a^3}{E\,I}\left(\frac{11p\,a}{120} - \frac{P}{3}\right) \qquad (8)$$

It is now proposed to write another expression for Θ as a function of the forces acting on the raised portion of the slab BC. Referring again to Fig. 4, let the origin of coordinates be taken at point C where the moment in the slab is zero, and let the positive x and y be to the right and upward respectively. Employing the expression for V from Eq. (1)

$$M_x = \left(\frac{w\,l}{2} + \frac{M}{l}\right)(l - x) - \frac{w\,(l - x)^2}{2} - M$$

which reduces to

$$M_x = \frac{w\,l\,x}{2} - \frac{M\,x}{l} - \frac{w\,x^2}{2} \quad \ldots\ldots (9)$$

and the equation of the elastic line of the portion of slab B-C is,

$$E\,I\,\frac{d^2\,y}{d\,x^2} = \frac{w\,l\,x}{2} - \frac{M\,x}{l} - \frac{w\,x^2}{2} \quad \ldots\ldots (10)$$

Let Eq. (10) be integrated once and the constant of integration be set equal to zero, since the slope is zero when x is zero, then

$$E\,I\,\frac{d\,y}{d\,x} = \frac{w\,l\,x^2}{4} - \frac{M\,x^2}{2l} - \frac{w\,x^3}{6} \quad \ldots\ldots (11)$$

Integration of Eq. (11) with the constant of integration again set equal to zero, since y equals zero when x equals zero, gives

$$E\,I\,y = \frac{w\,l\,x^3}{12} - \frac{M\,x^3}{6l} - \frac{w\,x^4}{24} \quad \ldots\ldots (12)$$

Now if x is set equal to l in Eq. (12), then y equals zero, which gives

$$0 = \frac{w\,l^4}{12} - \frac{M\,l^2}{6} - \frac{w\,l^4}{24}$$

in which M is simply a magnitude, and this reduces to

$$M = \frac{w\,l^2}{4} \quad \ldots\ldots (13)$$

Now substituting x equals l in Eq. (11), there is obtained an expression for the slope at B, which is

$$E\,I\,\Theta = \frac{w\,l^3}{4} - \frac{M\,l}{2} - \frac{w\,l^3}{6}$$

If the value of M from Eq. (13) is substituted and the resulting equation is solved for Θ, there is obtained

$$\Theta = -\frac{w\,l^3}{24E\,I} \quad \ldots\ldots (14)$$

Since the slab is continuous through point B, the slope Θ from Eq. (14) must equal the value of Θ from Eq. (8), which gives the relation

$$-\frac{w\,l^3}{24E\,I} = -\frac{p}{m\,a} - \frac{a^2}{E\,I}\left(\frac{11p\,a}{120} - \frac{P}{3}\right)$$

and if the value of p from Eq. (2) is substituted, there is obtained an expression for l in terms of a, as follows

$$\left(\frac{P}{3E\,I} - \frac{11P}{60E\,I} - \frac{11w\,l}{120E\,I}\right)a^2 - \left(\frac{w\,l}{m} + \frac{2P}{m}\right)\frac{1}{a^2} = -\frac{w\,l^3}{24E\,I}$$

which reduces to the following form that is more convenient for calculation

$$\frac{1}{3E\,I}(0.45P - 0.275w\,l)\,a^2 - \frac{1}{m}(w\,l + 2P)\frac{1}{a^2} = -\frac{w\,l^3}{24E\,I} \quad \ldots\ldots\ldots (15)$$

METHOD OF SOLUTION

For a given slab, soil constant, and edge load, the bending moment M may be obtained by trial. A value of l is assumed and the corresponding value of a is calculated from Eq. (15). If l has been assumed correctly, the value of M calculated from Eq. (3) will be equal to the value of M from Eq. (13).

EVALUATION OF THE SOIL CONSTANT

The soil constant m will be evaluated in terms of what is called the modulus of subgrade reaction, or simply the subgrade modulus, which is employed in rigid pavement design. This is defined as the load in psi that will produce a settlement of 1 in., based on a 0.1-in. or 0.05-in. settlement of a 30-in. diameter rigid circular bearing plate. Values of the subgrade modulus are stated to vary between 100 for a soft and plastic clay and silt, to 500 for an extremely hard gravel.*

Since p is the soil pressure per linear in. of a slab strip 12 in. wide, the pressure in psi is equal to $p/12$ and by direct proportion

$$\frac{p/12}{\text{subgrade modulus}} = \frac{e}{1}$$

from which

$$e = \frac{p}{12 \times \text{subgrade modulus}} = \frac{p}{m}$$

and therefore $m = 12 \times$ subgrade modulus

The objection may be raised that the pressure deformation characteristics of the soil as measured by the factor m are uncertain and cannot be closely determined from the subgrade modulus. However the following example

*Seelye, Elwyn E., *Data Book for Civil Engineers; Design*, John Wiley and Sons, Inc., New York, 2nd Edition, 1951, pp. 3–14.

indicates that a large variation in m has a relatively small effect on the bending moment in the slab.

EXAMPLE

Consider a typical wood frame school building comprised of a single row of classrooms and having an 8 ft wide outside corridor with cantilever roof. The weight under the front bearing wall at the edge of the foundation slab is 815 lb per linear ft, which is the edge load P. Let it be assumed that the foundation soil is a soft clay having a subgrade modulus of 100. The soil constant m is therefore 1200. The value of E may be assumed to be 2,000,000 psi.

If the slab thickness is taken as 6 in. then the moment of inertia I for a strip of slab 12 in. wide is 216, and the weight w of 1 in. length of this strip is 6.3 lb.

Let it be assumed that l equals 65.5 in. If this value of l and the above values of E, P, m, I, and w are submitted in Eq. (15), there is obtained

$$0.000000194\, a^3 - 1.70 \frac{1}{a^2} = -0.000170$$

from which, by trial, it is found that a equals 50.4 in. Now if these calculated values of a and l are substituted in Eq. (3) it is found that

$$M = \frac{50.4\,(413 - 815)}{3} = -6750 \text{ in.-lb}$$

and likewise from Eq. (13)

$$M = -\frac{6.3 \times 65.5^2}{4} = -6760 \text{ in.-lb}$$

The close agreement between the two calculated values of M indicates that the solution is correct. Generally a satisfactory agreement will result from not more than three or four trial values of l. It may be noted that quite large changes in l have only a slight effect on the value of a.

If the reinforcing is placed with its center 2 in. from the top of the slab and is stressed to 20,000 psi the required steel area is

$$A_s = \frac{6.75}{20 \times 0.88 \times 4} = 0.096 \text{ sq in.}$$

which would require No. 4 bars at 24 in. on centers.

The maximum soil pressure is determined from Eq. (2) and is found to be

$$p = \left(\frac{6.3 \times 65.5}{2} + 815\right) \frac{2}{50.4} = 40$$

Since this is the pressure per linear in. of slab strip 12 in. wide, the soil pressure per sq ft is $40 \times 12 = 480$ lb.

It should be noted that a uniform live load on the floor will not in itself produce bending or shear stress in the slab. However if the weight w in the analysis is increased to include a live load, it will be found that the moment M produced by the edge load is thereby reduced. Therefore live load should be neglected in the design of the slab.

LIMITATION OF THE ANALYSIS

The foregoing analysis is valid only if the total width of slab that is activated by the edge loads at the two opposite sides of the foundation is less than the width of the foundation. Otherwise the movement of the slab on one side is affected by the movement on the opposite side. In other words the point C of Fig. 1 does not remain on the x-axis and the moment at C may not be zero. In the above example the width of slab that is activated at the front of the foundation is $(l + a)$, which is about 9.5 ft. The width that is activated at the rear of the foundation is less than 9.5 ft, since the load is less without the corridor roof. Therefore somewhat more than 9 ft of the center portion of the 28 ft wide foundation is not affected by the edge loads and the solution is valid.

EFFECT OF VARYING *m* AND *E*

In the foregoing example if the value of the soil constant m is reduced 50 percent from 1200 to 600 and all other factors remain the same, the slab moment M becomes 7700 in. lb, an increase of only about 14 percent, and the soil pressure is reduced from 480 to 404 psf. On the other hand if the value of m is doubled from 1200 to 2400, the moment M is reduced about 13 percent to 5900 in.-lb, and the soil pressure is increased to 582 psf. Evidently the slab design should be based on a low value of the soil constant m, to allow for uncertainties in the pressure deformation characteristics of the soil. This procedure will have very little effect on the economy of the design.

The analysis is not sensitive to variations in the modulus of elasticity of the concrete. In the preceding example if the value of E is increased from 2,000,000 to 3,000,000, the moment M is increased from 6750 to 7300 in. lb, an increase of only 8.1 percent. A high modulus for design is therefore indicated.

SLAB DESIGN FOR BUILDINGS WITH BACK-TO-BACK CLASSROOMS

In the back-to-back arrangement of classrooms the foundation slab is loaded along the two opposite edges at the fronts of the classrooms and along the longitudinal center line by the bearing partition between classrooms.

The slab is designed for edge loads as described in the foregoing example. Marquardsen* has developed a method which may be used to determine the

*Marquardsen, R. P. V., "Practical Design of Thin Retaining-Wall Footings," ACI JOURNAL, Sept. 1952, *Proc.* V. 49, pp. 45-56.

bending moment in the slab due to the interior bearing partition. The edge moments are negative and produce tension in the top of the slab, while the center moment is positive with tension in the bottom. The top reinforcement is generally made continuous for the full width of foundation slab, to resist the negative moment at the edges and to control shrinkage and temperature cracks at other points. A reinforcing mat is then added at the bottom of the slab under the center partition, to resist positive moment at that point.

In this case the analysis is limited to the condition in which the total width of slab that is activated by the edge loads and the center load is less than the width of the foundation slab. This condition will always be fulfilled for a one-story wood frame building of this type.

Discussion of this paper should reach ACI headquarters in triplicate by July 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-50

Responsibilities of an Inspector*

By GRANT BLOODGOOD† and LEWIS H. TUTHILL‡

SYNOPSIS

Describes general principles, attitudes, and approaches appropriate for any inspector, on any size or kind of job. Place of the inspector in the construction engineer's organization is discussed with emphasis on cooperation with fellow inspectors, and the inspector's responsibility to carry out general organization policy whether it agrees with his personal judgment or not. How to maintain friendliness without familiarity keynotes section on relationship of inspector to contractor. Third section deals with direction of the activities of other inspectors.

INTRODUCTION

This paper was prepared as a lecture for Bureau of Reclamation inspectors attending concrete and earth control courses in February 1956. It is here published with minor changes under the sponsorship of ACI Committee 611—Inspection, in the interest of better inspection generally. Committee 611 recognizes that the inspector alone cannot perform effective inspection unless there is serious purpose and support of management behind him; another paper on this aspect of inspection is planned. However, the attitudes and responsibilities of the individual inspector are extremely important in successful inspection. In the following paragraphs, many of these problems of the inspector are discussed. The second person form used in the original lecture is retained for greater effectiveness.

INSPECTOR'S PLACE IN THE ORGANIZATION

First, what is your place in the construction engineer's organization? Usually, there will be your immediate superior, the chief inspector, and then the field engineer or resident engineer, in the line of authority between you and the construction engineer. On the large jobs, where more than one shift is worked and several inspectors are needed each shift, your immediate superior is usually a shift chief who in turn reports to the chief inspector. However, the smaller the job, the greater the responsibility that falls on the one or two inspectors employed.

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Your immediate superior will assign you to the area of concrete, earth, or other work you are to serve, will give you necessary instructions, and will visit you during the shift to see how things are going. During those visits problems and questions that arise on the work should be discussed. If possible they will be settled then and there, and you will take the indicated action. If they cannot be settled by him then, your supervisor will take matters up with his superiors at the first opportunity and the decision will be returned to you for your future information and guidance.

When a problem arises it is your responsibility to discuss the matter thoroughly with your chief so that he has a full and clear understanding of it. When the decision is reached, from whatever level it is handed down, it must be accepted by you and carried out, whether it agrees with your own judgment or not. This point is important—it is the very foundation of a smooth-working organization. Actually, these decisions, under the circumstances, will rarely be wrong. Chief inspectors and their superiors are usually men of long and wide experience; moreover, they usually have the advantage of a more comprehensive view and concept of all technical, administrative, and personal factors bearing on the matter; consequently, they usually reach a decision that is carefully and accurately balanced among these factors. However, if you feel that a wrong decision has been made through a lack of clear understanding of the matter at higher levels, and the question is of considerable importance, there is no reason why you cannot ask your immediate superior if he will arrange an on-the-job conference with those in authority.

If you so much as hint to the contractor's men that you are not in accord with a certain decision or instruction affecting the contractor, you will weaken the efficiency and usefulness of the inspection organization as well as your own standing with the contractor. You will find the contractor's men are usually quick to take advantage of such situations to break down standards set for the job; it is another example of the old principle of divide and conquer. None of you would willingly choose even for a moment to be a weak point in the united front your project construction force is working to create and must have to be most effective. By learning to take orders as well as give them, by diligently carrying out all instructions, and by making the organization objectives your objectives, experience is gained which develops in you the stature and caliber that is looked for when more responsible jobs are open.

You also have a responsibility to work and cooperate with other inspectors on similar work, particularly those who do your work on the preceding and following shifts. Every effort must be made to assure that the same interpretations and requirements are made on each shift. You should be ready to get on the job early enough and to stay long enough to become fully informed of current conditions and to see that the man who follows you is equally well informed. Needless to say you should not leave the job without advising your supervisor.

Reports and records of significant conversations with foremen and of instructions to them are an essential part of your duties. These reports are

often valuable in the preparation of findings of fact for defense of claims submitted by the contractor. For these reasons make your reports clear, accurate, complete, and neat. Use a sharp, No. 2 pencil. Do not make erasures; they seriously impair the value of a report as evidence in court. It is best simply to strike out errors, initial the strike-out, and continue.

RELATIONSHIP OF INSPECTOR TO CONTRACTOR

Your most important responsibilities of course are those connected with your relations with the contractor and his forces. They are quick to spot a phony but will accept a certain lack of experience if it is acknowledged. If you have a reasonable and practical viewpoint and approach, if they know you have taken the time and made the effort to become thoroughly familiar with specifications and instructions covering the work, and that your supervisor is working closely with you to teach you what is acceptable and what is not, some lack of experience should not be much to your disadvantage in getting along with the contractor's men on the job.

There are other attitudes you must develop, if you do not have them, in order to be most effective in your relations with the contractor's forces. It helps to be friendly, cheerful, and pleasant in your greetings and dealings each day. As you become fully conversant with the work there may be ways in which you can cooperate with foremen and make suggestions to the contractor's advantage without impairment of the client's interest. Such an attitude of helpfulness will make the other fellow much more inclined to meet requirements than will police-style enforcement.

Remember that a contractor is in business to make a profit; it is unfortunate for all concerned if he does not. Not only is it more difficult to get results wanted from him, but next time, if he is still in business, his bid will be higher and his experience, known in the trade, will tend to make others bid higher as well. So we are glad when our contractors make money. Competitive bidding tends to keep the profits from being excessive. Other things being equal, high profits make future bids lower by encouraging more competition. It should be no concern of yours how much profit a contractor is making. Under no circumstances should he be asked for more than specifications require merely because he is making a good profit.

Be fair and firm in your decisions and the contractor will respect you accordingly. Don't stall or pass the buck to your supervisor except when you have to get his help and advice. Give decisions promptly when you can. Be careful, however, not to promise or approve something unless you are sure your superiors will agree, as you will lose face if you have to take it back. Your superiors will do their best to back you up, so don't put them on the spot by agreeing to something that may be doubtful until you are sure where they stand.

By all means avoid unconstructive criticism of the contractor's plant, equipment, or methods, or any criticism of his personnel. All these are delicate subjects, particularly the latter. You can quickly become discredited,

and lose any influence you may have gained otherwise, by speaking to workmen and foremen unfavorably of other individuals in the contractor's forces. They naturally have an organizational loyalty, and such remarks will only be resented and detract from the usefulness and effectiveness of the one who makes them.

Usually a contractor bids on the basis of using certain equipment which he thinks will do the work acceptably. Sometimes part or all of it has been used before. Although you may be aware of equipment that you think will do the job better or even less expensively, remember that choice of equipment is the contractor's responsibility as long as it does work you can accept under the specifications. If there are ways the equipment can be used or modified at low cost to better advantage, these possibilities may be discussed with men in charge in a wholly impartial way for whatever help it may be, but clearly indicating that no such requirement is being made as long as results are acceptable.

Much the same applies to methods, but usually these are more easily changed. When their disadvantages are convincingly discussed in a friendly manner with those responsible for production quantities and acceptability of results, mutually advantageous modifications can often be worked out. In all such conversations you should carefully avoid instructing or seeming to require that things be changed. These should never be more than suggestions placed before the contractor for his consideration; otherwise he is likely to disclaim responsibility for results and may even put in a claim for added cost. The same applies to instruction of workmen. Make your requirements known to the foreman, not the workmen. Then the contractor cannot disclaim responsibility because you were running the job. Although you should not act as the contractor's foreman, if you see the contractor planning certain operations that you know will lead to pitfalls, you have an obligation to call it to the attention of the contractor or his superintendent.

Your responsibility is to see that proper results are obtained hour-by-hour, batch-by-batch, load-by-load, whatever the work may be. If, despite your good efforts and attempts to work effectively with the contractor's forces, his methods, equipment, or personnel are unable or unwilling to produce proper results, firm action must be taken. Fortunately for you, you do not have to bear alone the responsibility of getting something constructive done in a case like this. If the work of inferior and unacceptable quality persists, this will have developed gradually, not in the last 5 minutes, and therefore you will have talked it over with your chief, and he probably with his superiors, several times before. Together a line of action will doubtless have been formulated, and probably whatever ultimatum has to be issued to the contractor will be given to him by someone higher than you in the construction engineer's organization, perhaps by the construction engineer himself.

Better work is easier to obtain when friendly, cooperative relations are created and maintained with the contractor's forces. You should realize that after a contract is signed the contractor and his forces are working for

the client too. Their objectives and yours are essentially the same: to build a good job. Despite the profit motive on his side, a good contractor and his forces also take natural pride in a good job well done. They are also human and will note and resent any attitude implying that they are less than an equal with the client's forces in getting the job done. This can be avoided and good feeling engendered instead by a conscious effort to do and say rightly many little things. Ask their opinion when you can. Use their suggestions when possible and let them know their opinion is valued and respected. Include their people in social events. Invite their key men to job training sessions. Many small ways and opportunities to get this idea across will appear if you are looking for them. Small gestures of this kind are far more effective than any one speech or event.

In fostering these favorable relationships certain precautions are necessary. Friendliness should be without familiarity. It is said that familiarity may breed contempt. Do not let this happen. It would impair your effectiveness and dignified standing as a representative of the client. You should not take part in drinking and poker parties with foremen and workmen you are dealing with on the job. Moreover, you should not accept gratuities at the contractor's expense. These actions, if engaged in or permitted by you, will inevitably inhibit to some degree your decisiveness of action in cases where some decision must be made contrary to the contractor's wishes.

DIRECTION OF OTHER INSPECTORS

Some of you are inspectors and others of you will doubtless some day also be supervising inspectors. Therefore it is appropriate to mention here several ways in which you are responsible to the men working under your direction:

1. To outline in advance those matters for which each inspector in your charge will be directly responsible.

2. To inform your inspectors as to decisions which they should refer to you.

3. To see that your inspectors have access to and read all necessary instructions pertaining to their work, including letters of instruction and laboratory reports concerning the work, and information concerning special problems and conditions which are apt to arise.

4. To see that an inspector is not placed on the job without knowledge of special arrangements or agreements that have been made with the contractor and that affect the work with which the inspector is concerned. Particularly, the inspector should be informed at once of any concessions or special interpretations of the specifications.

5. To see that your inspectors are made aware that they have your full support in all proper execution of the inspection work.

6. Whenever practicable (and it is seldom that it is not), to obtain the opinions of inspectors and engineers close to the job before reaching decisions relative to requests and representations made by the contractor.

7. To see that every question concerning the job, even though sometimes not pertinent, is promptly and fully answered, and that you make your inspectors feel free to ask such questions.

Also, you will find it worthwhile to have meetings occasionally of inspectors and supervisors for open discussion of job problems, practices, and requirements, and I urge that you establish the practice of having such meetings if it is not now your practice to do so. You will find that a few hours cannot be spent more rewardingly personally, or with more value to the job, than those spent in instruction of your inspectors.

Discussion of this paper should reach ACI headquarters in triplicate by July 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

CONCRETE BRIEFS

Notes from Field and Office

More on Winter Concrete 905

More on Winter Concrete

By LEWIS H. TUTHILL*

At the ACI regional meeting in Montreal (Oct. 25, 1956) one session was devoted to a panel discussion of winter concreting practices in general and, in particular, to those recently adopted by ACI as its standard "Recommended Practice for Winter Concreting (ACI 604-56)." In this session and elsewhere during these meetings, it became evident that it would have been helpful if the recommendation could have included more in the text on the amount and sufficiency of strength attained at the end of recommended periods of protection.

Strength potentials and time-strength requirements of concrete vary widely. Beyond what is shown in the nine figures and references of the appendix of the standard, it was regarded as impractical and unnecessary to attempt to include information on strength. In respect to strength, winter concrete is the same as other concrete in that it behooves the inspector and the builder to make sure that enough strength has developed to more than meet their demands before placing a burden on the concrete. In winter, as in other seasons, sufficient time and favorable environment must be provided for development of the strength required for the purpose. Although time and environment necessary for all concretes to reach a certain strength will vary widely, they do not vary widely in what is necessary to assure durability of all concretes placed in winter; hence it was practical to make concise recommendations for good winter concreting practice to assure durability.

Since the prime objective of the recommendation was to describe means of assuring durability of winter-placed concrete, information on strength at low temperatures was left largely to the figures in the appendix, in which considerable data are provided. However, on examining these with this particular question in mind, it is noted that most of the figures give strength of concretes initially stored at low temperatures, then followed by storage at higher temperatures until tested. An exception is Fig. 6 which gives the strength of good-quality, air-entrained concrete with W/C at 0.50 and Type II cement, after initial periods of storage at 50 F as recommended for protection and good durability, and followed by periods of storage at low temperatures. This could well represent a winter concrete protected as recommended in ACI 604-56.

For example, this concrete with the recommended 1 percent of calcium chloride (Block E, Fig. 6), attained 1500 psi during the initial 3-day period of protection at 50 F, an ample strength to sustain most construction requirements except for unsupported slabs and beams, and floor slabs subject to immediate abrasion. Moreover, when then exposed to 32 F it continued to gain strength, although more slowly than at higher temperatures, and reached a strength of 2600 psi at 7 days, 3000 psi at 12 days, and 3800 psi at 28 days. With Type I cement, somewhat higher strengths would be obtained at the earlier ages. Freezing after protection of course stopped strength development (Block F, Fig. 6), but when temperature was raised

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TABLE 1—EFFECT OF AIR ENTRAINMENT ON CONCRETE STRENGTH AND DURABILITY

W/C ratio	Durability, in terms of number of freezing and thawing cycles withstood		Strength at 28 days, psi	
	Plain	Air-entrained	Plain	Air-entrained
0.46	158	984 no failure* Modulus 85%	4280	3470
0.51	130	984 no failure* Modulus 87%	4370	3310
0.57	68	984 no failure* Modulus 88%	3800	3230
0.63	34	984 no failure* Modulus 73%	3540	2580
0.68	20	844	3500	2560
0.86	16	865	2630	1960

*Failure is defined as the point at which sonic modulus of elasticity falls below 60 percent of that at zero cycles.

again to only 50 F, normal strength development ensued.

Also at the regional meeting it was learned that a misconception apparently existed in some quarters that air entrainment was not beneficial in improving the durability of the better structural concretes, and since it did not serve this purpose, it was best not used because it reduced strength a few hundred psi. It is true that at the same W/C (with reduced cement content for the same slump) air entrainment does reduce strength a few hundred psi, but it is not true that it does not materially improve durability of all classes of concrete, as indicated by freezing and thawing tests.

A most recent and direct group of test data showing this was presented at the Montreal meeting by J. C. Kingston, concrete control engineer for the Montreal portion of the St. Lawrence Seaway, in his paper "Selection of Concrete Aggregates for the St. Lawrence Seaway Locks." With Mr. Kingston's permission these pertinent data are summarized in Table 1, which clearly shows that in all qualities of concrete, as indicated by the range in W/C, resistance to laboratory cycles of freezing and thawing is greatly increased by air entrainment with proven air-entraining agents.

In terms of extra life and serviceability, is not the indicated strength loss of the few hundred psi a small price, easily afforded or compensated for, to gain such several-fold improvements in durability?

For interior concrete not exposed to wet freezing and thawing, indicated strength loss may be regarded by some as unworthy of the other benefits imparted by air entrainment. For such concrete, by reducing air content about one-fourth from that indicated by "Recommended Practice for Selecting Proportions for Concrete (ACI 613-54)," loss of strength is lessened considerably yet the benefits of air entrainment in improving workability, and in reducing segregation, sand and water requirement, and shrinkage remain largely effective. It may be safely concluded that the correct amount of entrained air, entrained with a suitable agent, is beneficial and worthwhile in most concretes.

COMING UP

Scheduled for early publication in "Concrete Briefs" is Manuel Savran's report on "Column Design for Combined Bending and Axial Load —Uncracked Sections."

"Recommendations of the International Subcommittee on Concrete for Large Dams on the Definition of Concrete Mixes Used in Dam Construction," is also scheduled for a forthcoming issue.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Bridges

First French large railroad bridge in prestressed concrete: La Voulte viaduct (in French)

L. Carpentier, *Travaux* (Paris), V. 40, No. 263, Sept. 1956, pp. 487-496

Reviewed by M. W. Jackson

The first large railroad bridge in France in prestressed concrete is across the Rhone, south of Valence. It consists of five spans, each of about 180 ft. Because of the importance of the bridge, the prestressing was carried out with all possible checks. The ultimate factor of safety was arranged in such a way that the structure provides guarantees comparable with those of a reinforced concrete bridge.

The cables were of a type not before used in France; they were anchored by "button heads" on end-plates and were tensioned either by direct pull or helical displacement.

Prestressed concrete highway bridge in South Africa: Ellis Brown viaduct (Vejbro i forspaendt beton i Sydafrika)

K. H. Sorensen, *Ingenioren* (Copenhagen), V. 65, No. 40, Oct. 6, 1956, pp. 800-806

Reviewed by Jesper Strandgaard

A detailed description of a 1500 ft long highway bridge over the Umgeni River near Durban, South Africa. The bridge consists of fifteen 100-ft spans, each span of 14 precast, post-tensioned (Freyssinet system), simply supported concrete girders, and a parabolic facial girder on each side for appearance. Steel forms were used for the precast girders. The cross section is 71 ft 10 in. wide between the precast concrete handrails, with two 24-ft roadways, a 3-ft divider, two 6-ft bicycle paths, and two 4 ft 11 in. sidewalks. The piers are on concrete piles. Expansion bearings are precast concrete rockers of Freyssinet type. Bridge was designed in accordance with "British Standard Specifications for Highway Bridges," and based on a 28-day cube strength of concrete of at least 6000 psi, but governing for the mix was a desired strength of 4500 psi after 5 days, necessary for tensioning of the cables. The obtained average 28-day cube strength was 9000 psi. Cost of the bridge, built in 1954-55, not including the approaches, was approximately $1.2 million.

Prestressed concrete bridges

D. H. Lee, *Chartered Civil Engineer* (London), Sept. 1956, pp. 10-14

Discusses the limits of economy for prestressed bridges as illustrated by long-span structures in many countries. The tentative conclusion is that the maximum economic span for prestressed rural bridges is somewhat in excess of 150 ft but that the steel truss bridge economy for longer spans will vary somewhat with the location.

New 153-m reinforced concrete bridge at Treguier (Le nouveau pont en beton armé de 153 m de portée à Tréguier)

E. Bideau and E. Lacombe, *Annales de L'Institut Technique du Batiment et des Travaux Publics* (Paris), No. 90, June 1955

Reviewed by Phillip L. Melville

A description of a twin reinforced concrete arch bridge with a 500-ft span and 85-ft rise with a suspended unbraced deck. Reinforcement was copper chromium steel. Hangers were bitumen and glass protected prestressing wires.

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For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the "Current Reviews" section are available from ACI headquarters on request.

Construction

Mass production for eight apartments a day in the Parisian district (Fabrication industrielle de huit logement par jour dans la region parisienne)

R. CAMUS, *Annales de l'Institut Technique de Bâtiment et des Travaux Publics* (Paris), No. 101, May 1956. Series: Technique General Construction, No. 17, 28 pp.

Reviewed by HENRI PERRIN

Describes the organization of plants precasting concrete panels used for the erection of six- to eight-story houses. The panels, cast horizontally and vertically, weigh up to 7.5 tons and contain floor tiling, metal doors and windows, ducts, etc., thus reducing erection time to a minimum. Data about the concrete used, its strength, and the curing process are given. Labor required is indicated. Numerous photographs illustrate each step from the plant itself to the final stage.

Construction with large-size building elements (Bauen mit grossformatigen Bauelementen)

WALTER KOSACK, *Der Bauingenieur* (Berlin), V. 31, No. 2, Feb. 1956, pp. 67-69

Reviewed by ARON L. MIRSKY

Describes and pictures wall construction using king-size (approximately 40 in. long x 33 in. high x 10–12-in. thick) block. Walls are thus one block thick, but blocks of "open-pore" concrete are so shaped that vertical joints are offset. Handling and placing of block is by crane, using tongs.

Dams

Echelles D'Annibal Dam at Aigueblanche (in French)

X. RACT-MADOUX and M. BOUVARD, *Travaux* (Paris), V. 40, No. 262, Aug. 1956, pp. 447-460

AUTHORS' SUMMARY

Discusses certain special features relating to the construction of the intake dam of the Isere-Arc waterfall. This dam is noteworthy for its arch design, which has enabled rock supports to be found, either side of a gorge, below an alluvium covering a 100 m in thickness.

Apart from the calculations relating to the foundations, the authors draw special attention to the position of construction joints in the structure, design of the concrete mask holding back the possible leakages at the bottom, and the choice of the valving gear in relation to the evacuation of solid matter.

With regard to aesthetic considerations, it was found possible to keep all the valve mechanism chambers below the general horizon line of the dam, except for the central control room on the left bank.

Concrete dams (Beton-Staumauern)

F. TOELKE, *VDI Zeitschrift* (Duesseldorf), V. 98, No. 16, June 1, 1956, pp. 901-903

Reviewed by ARON L. MIRSKY

Annual review, summarizing briefly developments (primarily European) in such topics as intrinsic stresses ("Eigenspannungen") (heat of hydration, shrinkage, creep), frost-resistant concrete, concrete production, and foundation sealing.

Earth and stone dams are covered in other reviews in the same issue.

Design

Group of tanks at Perpignan (Groupe de reservoirs "du Cimetiere de l'Quest" à Perpignan)

R. DEVARS DU MAYNE, *Annales de L'Institut Technique du Bâtiment et des Travaux Publics* (Paris), No. 90, June 1955

Reviewed by PHILLIP L. MELVILLE

Description of design and construction of three water tanks (two buried and one elevated). Jointless reinforced concrete was used.

Structural air-raid protection—shelter construction (Baulicher Luftschutz—Schutzraumbau)

WIENDIECK, *Die Bauzeitung* (Stuttgart), V. 61, No. 4, Apr. 1956, pp. 152-154

Reviewed by ARON L. MIRSKY

Another article dealing with the world's current preoccupation: preservation of the (often) not-so-innocent bystanders.

Materials

ASTM standards on mineral aggregates and concrete (with selected highway materials)

American Society for Testing Materials, Philadelphia, 1956, 360 pp., $3.75

This compilation of ASTM standards includes the latest editions of standard and tentative specifications, test methods, and definitions of terms pertaining to mineral aggregates, concrete, and selected highway

materials. The publication is sponsored by Committee C-9 on Concrete and Concrete Aggregates and Committee D-4 on Road and Paving Materials, and includes all standards for concrete and concrete aggregates prepared by Committee C-9, but only those for selected nonbituminous highway materials or those involving aggregate gradings or tests developed by Committee D-4. It also contains pertinent specifications for cement under the jurisdiction of Committee C-1 on Cement, and references specifications on concrete reinforcement bars prepared by Committee A-1 on Steel.

The book contains 98 standards in their latest forms—56 test methods, 33 specifications, 7 definitions of terms, and 2 recommended practices. Subjects covered include specifications and methods of test for aggregates (crushed stone, crushed slag, gravel, lightweight); ready-mixed concrete; air-entraining admixtures; methods of test for air content of concrete, compressive strength, flexural strength, freezing and thawing, slump test; brick and block pavement materials; concrete curing material; expansion joint fillers; cement.

Evolution of the French cement industry on the technical and research plan (Evolution de l'industrie cimentière Française sur le plan des recherches et sur le plan technique)

H. Lafuma, *Revue des Matériaux de Construction* (Paris), No. 482-483, 1955, pp. 39-47
Reviewed by Phillip L. Melville

A review of technical progress in the manufacture of hydraulic binders with cements in the lead. Progress in equipment and processes are discussed. Evolution of the chemical and physical properties of cements in France and in relation to United States practices are analyzed.

Evolution of the French cement industry of the professional and economic plan (l'evolution de l'industrie cimentière Française sur le plan professionnel et economique)

R. Farre, *Revue des Matériaux de Construction* (Paris), No. 482-483, 1955, pp. 23-38
Reviewed by Phillip L. Melville

A historical description of the cement industry in France since its birth a century ago. Production, distribution, economy, and business administration are studied on a nation-wide basis.

Use of granulated slag cements and crushed slags in road and airport pavements in Belgium (Utilisation des ciments a base de laitier granule et laitier concasse dans les revetements routiers et pistes d'envol en Belgique)

L. Blondiau, *Revue des Matériaux de Construction* (Paris), No. 487, Apr. 1956, pp. 83-94, and No. 488, May 1956, pp. 109-126
Reviewed by Phillip L. Melville

A study of the use of slag cements to obtain quality pavement concrete. Laboratory and field tests were performed. It is concluded that blast furnace slag cements give lower compressive but higher flexural strength than portland cements. Slag as an aggregate is critical for its porosity, density, and adhesion. It is recommended that the slag be compounded according to Parker, be cooled quickly from high temperatures to prevent the formation of unstable dicalcium silicate, be dense, be aged at least 2 weeks, be cooled to 15 to 20 C, and be homogeneous. From tests, it is concluded that with proper care slag concrete will be equal to portland cement concrete containing igneous aggregates.

Manufacture of hydraulic lime (fabrication de la chaux, hydraulique)

J. Juhasz, *Revue des Matériaux de Construction* (Paris), No. 490-491, July-Aug. 1956, pp. 178-180
Reviewed by Phillip L. Melville

A study of the manufacture of hydraulic lime to obtain a product which can challenge portland cement in economy and quality.

Prestressed Concrete

Test of a model of flat prestressed concrete slab (Comple rendu des essais effectués sur une maquette de plancher dalle en béton précontraint, sans nervures ni champignons)

P. Lebelle, *Annales de L'Institut Technique du Batiment et des Travaux Publics* (Paris), No. 90, June 1955
Reviewed by Phillip L. Melville

A quarter-scale model of a flat slab resting on six columns without drop panel was tested to failure. Prestressing was in two directions and the load uniform and equally distributed. The conclusions show that the deflection-rupture load of such construction

can be forecast, with a safety factor relative to the overload alone of about 2.5 where there are neither shocks nor vibrations. It was not possible to evaluate the resistance to punching.

Prestressed concrete supports for electric lines (in French)

P. LOMBARD, *Travaux* (Paris), V. 40, No. 255, Jan. 1956, pp. 23-29

Reviewed by M. W. JACKSON

Development and use of tapered, hollow rectangular "poles" for high tension lines is described. Heights up to about 60 ft are used. These replace the usual steel towers. In Algeria in 12 years, more than 37,000 of these have been placed in service.

Precast concrete warehouses

Engineering (London), V. 181, No. 4709, June 8, 1956, pp. 476-477

Reviewed by ARON L. MIRSKY

Description of Dow-Mac system employing prestressed precast members. Columns may be from 15 to 50 ft long; main beams up to 80 ft clear span; and resulting frames may be spaced up to 30 ft on centers. Prestressed cellular (waffle) slabs have been successfully used as siding.

Properties of Concrete

Evolution of concrete technology (L'evolution de la technology du béton)

R. L'HERMITE, *Revue des Matériaux de Construction* (Paris), No. 482-483, 1955, pp. 50-56

Reviewed by PHILLIP L. MELVILLE

The main phases of concrete making are discussed. Starting with composition, gradation according to Feret, Bolomey, Fuller, Caquot, Faury, Valette are studied. It is important for optimum results, to take advantage of vibration (at frequencies of the order of 12,000 cps), certain admixtures (such as air-entraining agents and dispersents), drying (by vacuum pressure, centrifugal forces, or laminating), injections by activated colloidal mortar. Progress has also been reported in cold weather concrete mixing and transporting, and forming. Through scientific research many properties of concrete are now better understood: volume changes can be controlled and the plastic/elastic changes are used in design. Sonic and radioactive methods of investigation permit determining properties of concrete in place.

Compressive strength of concrete. Its measure with the Schmidt test hammer (La resistance à la compression du béton. Sa mesure par le scleromêtre Schmidt)

J. CHEFDEVILLE, *Annales de L'Institut Technique du Batiment et des Travaux Publics* (Paris), No. 95, Nov. 1955, Series: Test and Measurement, No. 34, 44 pp.

Reviewed by PHILLIP L. MELVILLE

A series of tests were made with the Schmidt impact hammer on sundry concrete samples made with different types of cements. Testing conditions were varied (specimens in compression in a testing machine, resting on two points, resting on foam rubber, resting on a concrete floor). Results analyzed from a statistical point of view and compared to Swiss data. Scope of the test is given as very limited.

Capillarity in fabricated concretes (La capillarité des bétons manufacturés)

L. MARILL, *Publication Technique* No. 75, Centre d'Etudes et de Recherches de l'Industrie des Liants, Hydrauliques, Paris, 23 pp. (extract from *Revue des Matériaux de Construction*, No. 485, Feb. 1956)

AUTHOR'S SUMMARY

Deals with the main causes of capillarity in fabricated concretes: water content, granular composition, hardening conditions, and concludes that capillarity is mostly due to low water content.

The action of water repellents is revealed by comparative determinations between water-repellent concretes and those without admixture. Protection with a fluoride-, or chloridized rubber-base coat, or by impregnation, enable appreciably reducing water penetration.

Reduction of shrinkage and creep of concrete by curing in steam and hot air (Verringern des Schwindens und des Kriechens von Beton durch Haerten in Dampf und Heissluft)

F. BEER, *VDI Zeitschrift* (Duesseldorf), V. 98, No. 20, July 11, 1956, p. 1062

Reviewed by ARON L. MIRSKY

An abstract of a paper, "Important Reduction of Shrinkage and Creep of Concrete Elements" (in Hungarian), by G. Balazs and J. Kilian, in *Wissenschaftlichen Mitteilungen der Technischen Hochschule fuer Bauwesen in Budapest*, V. 1, No. 3, 1955, pp. 65-76.

Long-term observations indicated that curing by high-pressure (around 8 atmosphere) steam followed by hot air reduced 400-day

shrinkage considerably; certain combinations even yielded a *negative* shrinkage (*i.e.*, specimen length increased). Low pressure (atmospheric) steam and hot air curing showed similar effects. Creep (sensibly static at 600 days) was also reduced.

Application to prestressed structures is noted.

Shrinkage and cracking of cements and concretes (Retrait et fissuration des ciments et bétons)

H. Lafuma, *Revue des Matériaux de Construction* (Paris), No. 489, June 1956, pp. 145-147
Reviewed by Phillip L. Melville

Neat cement will shrink 2 or 3 times more than 1:3 mortar and 4 to 6 times more than concrete. High shrinkage results from modern requirements for high-early-strength which are obtained by using more calcareous mixtures and finer grindings.

Structural Research

Model investigation and massive construction (Modellversuch und Massivbau)

P. Lardy, *Der Bauingenieur* (Berlin), V. 30, No. 10, Oct. 1955, pp. 360-363
Reviewed by Aron L. Mirsky

A short but trenchant résumé of the purpose and results of what is perhaps better known as "experimental stress analysis." The use of this powerful method in investigating the "play of forces" (*Kraeftespiel*) and the interaction of various members, using three-dimensional models, is especially emphasized with photographic examples from Swiss investigations.

Measurement of stress in plain and reinforced concrete (Spannungsmessung im Beton und Stahlbeton)

K. Kammueller, *Der Bauingenieur* (Berlin), V. 31, No. 2, Feb. 1956, pp. 47-48
Reviewed by Aron L. Mirsky

Two improved instruments are described, one a Carlson-like stress gage designed to be embedded in concrete, and the other a mechanical-linkage type designed to be attached to the outside of a member, like the Whittemore or Huggenberger gages.

First instrument measures the pressure in a fluid (mercury) contained in a hollow disk embedded in the concrete. Apparatus is so constructed that it is affected only by dimensional changes due to stress; changes due to creep, shrinkage, etc., are self-compensating. Theoretical proof of operation is given, as well as several comparisons of results using this pressure meter and a strain meter.

Second apparatus, said to have little backlash, uses two initially-buckled strips. These undergo increased or decreased buckling as load is increased or reduced; their bending may be measured by various means.

On the interaction of reinforcement and concrete in reinforced concrete structures (Ueber das Zusammenwirken von Bewehrung und Beton in Stahlbetonbauwerken)

E. Amstutz, *Der Bauingenieur* (Berlin), V. 30, No. 10, Oct. 1955, pp. 353-359
Reviewed by Aron L. Mirsky

Summarizes Swiss investigations on bond and shear cracking. Course of investigations followed the classic path, pull-out tests to beams. Strains in reinforcement were measured with electric strain gages; external extensometers and dial gages were also used.

Uniformly distributed loading for laboratory tests

J. S. Caswell, *The Engineer* (London), V. 201, No. 5238, June 15, 1956, pp. 669-670
Reviewed by Aron L. Mirsky

Description of versatile hydraulic loader interposed between beam or slab under test and platen of testing machine to obtain good approximation of uniformly distributed loading.

Beam test shows need for web steel

W. E. Dean, *Engineering News-Record*, V. 157, No. 25, Dec. 20, 1956, p. 36-37.

Test on full-size precast, pre-tensioned, prestressed concrete beams composite with cast-in-place deck was required by Florida State Highway Department to justify use of these beams for bridges. Detailed results are given, indicating that bond in such construction is achieved without shear keys, by tie-down stirrups only, and that some minimum web reinforcement is advisable even where computations indicate it is not needed.

Resonance stress testing

B. Bramall, *The Engineer*, (London), V. 202, No. 5247, Aug. 17, 1956, pp. 224-228
Reviewed by Aron L. Mirsky

Valuable paper on a method for determining existing longitudinal loads (forces) in

members of structures. Such loads (forces) may be due to dead weight, to prestressing, or to constraints (*i.e.*, redundancy, especially where there is interaction between bracing and other secondary members and main structural members). Method used is to establish transverse vibrations in the member via external excitation and then measure the response at the quarter points. (In effect, member takes the place of the vibrating wire used in some strain gages.) Theory, application to various special cases, and strain-gage circuitry are discussed; two numerical examples are also given.

For discussion involving inclusion of effect of shear deflection, see title source, Sept. 21, p. 402, and Oct. 26, p. 582.

Savings to be effected by the more rational design of cased stanchions as a result of recent full-size tests

OSCAR FABER, *The Structural Engineer* (London), V. 34, No. 3, Mar. 1956, pp. 88-109

Reviewed by C. P. SEISS

Tests under axial load of 19 full-size columns consisting of rolled steel sections encased in concrete and representative of "combination columns" of ACI Code. Includes full report of test procedures and results, and discusses design loads on basis of present British code and proposed new design procedure.

General

Safety of structures (in French)

R. LEVI, *Travaux* (Paris), V. 40, No. 262, Aug. 1956, pp. 435-440

Reviewed by M. W. JACKSON

The philosophy of the "factor of safety" is discussed. An hypothesis is considered that the variables upon which depend the insecurity of a structure do not follow the normal law of probability but that their logarithms do. An over-all factor of safety is defined by a formula which simultaneously takes into account all the standard deviations by which the various gaussian logarithms are governed.

Discussion on the report on structural safety

The Structural Engineer (London), V. 34, No. 9, Sept. 1956, pp. 307-323

Discussions, oral and written, of committee report published in *The Structural Engineer*, V. 33, No. 5, May 1955, p. 141. (See "Current Reviews," ACI JOURNAL, Jan. 1956, *Proc.* V. 52, p. 594.)

Cut-offs in modern hydraulic diversion works on alluvial rivers

SERGE LELIAVSKY, *The Engineer* (London), V. 202: No. 5246, Aug. 10, 1956, pp. 184-188; No. 5247, Aug. 17, 1956, pp. 228-230

Reviewed by ARON L. MIRSKY

Discussion of various types of cutoff walls used with barrages, etc. Several types of concrete sheet piles are included.

Evolution of the manufactured concrete industry (L'evolution de l'industrie du béton manufacturé)

L. MARILL, *Revue des Matériaux de Construction* (Paris), No. 482-483, 1955, pp. 57-63

Reviewed by PHILLIP L. MELVILLE

A comprehensive study of the ready-mixed and transit-mixed concrete industry in France and that of the much more developed precast concrete industry.

New socket joint for reinforced concrete pressure pipe (Eine neuartige Muffenverbindung fuer Stahlbeton-Druckrohre)

HELMUT CARP, *Der Bauingenieur* (Berlin), V. 31, No. 2, Feb. 1956, pp. 49-52

Reviewed by ARON L. MIRSKY

Describes a relatively simple joint which requires only a slotted rubber ring and accommodates large movements (36 mm either side of "normal"). Pipe with 1000 mm diameter was used in one line of the Gladbeck-Hahnenbach pumping system, which lifts water from the Hahnenbach into the Boye; results showed joint to be watertight under pressure, but not suited for reduced pressures accompanying water hammer or shock waves.

Arrangement of groins on a sandy beach

S. NAGAI, *Proceedings*, ASCE, V. 82, WW4, Sept. 1956, pp. 1063-1 to 1063-13

Discusses the interesting problem of arranging groins effectively for protection against erosion by wave action on a sandy coast. Presents the relationship between the length of groins, spacing, and orientation with respect to the shore line, the direction of wave propagation, and the breaking point of the waves. The relationship between wave steepness and sand transport and some experimental results on special types of groins are presented. Model studies and theoretical analyses are illustrated.